Crises: Equilibrium Shifts and Large Shocks

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Cowles Lunch Talk
February 2018
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Widely credited with having shifted the Eurozone economy from a "bad equilibrium" (high sovereign debt spreads and growing fiscal deficits mutually reinforcing each other); to a "good equilibrium" (with low spreads and sustainable fiscal policy).
explaining equilibrium shifts

in many economic (and other) settings...

- have convincing explanations/models of strategic complementarities giving rise to self-fulfilling outcomes
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e.g., sovereign debt markets, financial crises, revolutions
• Consider a setting where...

Levels and Changes
Consider a setting where...

- a coordination game is played every period whose payoffs depend on a "fundamental state"
Consider a setting where...

- a coordination game is played every period whose payoffs depend on a "fundamental state"
- the fundamental state evolves according to an exogenous random process

We ask: which informational events (must) trigger equilibrium switches? We identify two distinct scenarios must trigger equilibrium shifts:

1. Fundamentals hit a critical boundary (we will see how this boundary is determined)
2. There is a large enough shock to fundamentals - even if the shock does not take us to the critical boundary (we will see how big this jump must be)

We explain when shifts must occur but allow for multiplicity and hysteresis in many scenarios
Consider a setting where...

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- there is *incomplete information* about innovations to fundamentals...
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• Key strategic implication:
  • with no or small shocks, can keep doing same thing as before because you may rationally be confident that others will do so
  • with large shocks,
    • not rational for marginal player to be confident of others’ behavior; uniform rank beliefs select "risk dominant" equilibrium
Both levels and change predict shifts.
1. Both levels and change predict shifts.
2. Don’t always play risk dominant equilibrium. but switches only to risk dominant equilibrium.
Part 1 (Analysis): Individual Rationalizable Behavior in a Static Coordination Game with Incomplete Information

- Carlsson and van Damme 93 "global game" model
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- our main large shock result relies on fat tails (c.f., large normal prior, normal noise global game literature)
Part 2 (Interpretation): Aggregate Behavior in Dynamic Coordination Game

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Part 2 (Interpretation): Aggregate Behavior in Dynamic Coordination Game

- Static coordination game played repeatedly under evolving fundamentals and fat-tailed prior on common innovations
- Assume hysteresis: follow majority play from previous period if rationalizable, otherwise
- Majority behavior switches in response to either extreme enough level of fundamentals or a large shock
Complete Information Game

- a continuum of players
Complete Information Game

- a continuum of players
- each player decides to "invest" or "not invest"
Complete Information Game

- a continuum of players
- each player decides to "invest" or "not invest"
- "return to investing" $x$

formally, payoff to not investing is 0 and payoff to investing is $x + 1$, where $x$ is the proportion of other players investing.
Complete Information Game

- a continuum of players
- each player decides to "invest" or "not invest"
- "return to investing" $x$
- invest if the return exceeds the expected proportion of others not investing

\[ \text{payoff to investing} = x + 1, \text{ where } x \text{ is the proportion of others not investing} \]
Complete Information Game

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- "return to investing" $x$
- invest if the return exceeds the expected proportion of others not investing
- formally, payoff to not investing is 0 and payoff to investing is $x + \alpha - 1$, where $\alpha$ is the proportion of other players investing
Equilibria…
Equilibria...

- if $x > 1$, players have a dominant strategy to invest
• Equilibria...
  • if $x > 1$, players have a dominant strategy to invest
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Complete Information Game Equilibria

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Terminology: the *risk dominant* action is the one that would be chosen by a player with a uniform belief over the proportion of others who will invest.....
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• **Terminology:** The *risk dominant* action is the one that would be chosen by a player with a uniform belief over the proportion of others who will invest.....
  - if \( x > \frac{1}{2} \), "all invest" is the risk dominant equilibrium
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- if \( x > \frac{1}{2} \), "all invest" is the risk dominant equilibrium
- if \( x < \frac{1}{2} \), "all not invest" is the risk dominant equilibrium
• common prior mean return is $y$
• agent $i$ has return to investment is $x_i = y + \sigma z_i$ where
  • parameter $\sigma > 0$ measures "shock sensitivity"
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  • agent $i$'s shock $z_i$ has two components, $z_i = \eta + \varepsilon_i$
Incomplete Information / Heterogeneous Returns

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  - parameter $\sigma > 0$ measures "shock sensitivity"
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    - a common shock $\eta$
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- parameter $\sigma > 0$ measures "shock sensitivity"
- agent $i$'s shock $z_i$ has two components, $z_i = \eta + \varepsilon_i$
  - a common shock $\eta$
  - an idiosyncratic shock $\varepsilon_i$
Maintained Assumptions about Shocks

1 thick tailed common shocks: \( \eta \) is distributed according to density \( g \) with thick (regularly varying) tails, i.e.,

\[
\lim_{\lambda \to \infty} \frac{g(\lambda \eta)}{g(\lambda \eta')} \in (0, \infty) \text{ for all } \eta, \eta'
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Maintained Assumptions about Shocks

1. **thick tailed common shocks**: $\eta$ is distributed according to density $g$ with thick (regularly varying) tails, i.e.,

$$\lim_{\lambda \to \infty} \frac{g(\lambda \eta)}{g(\lambda \eta')} \in (0, \infty) \text{ for all } \eta, \eta'$$

2. **thinner tailed idiosyncratic shocks**: $\varepsilon$ is distributed according to log-concave density $f$ (i.e., log $f$ is concave)
Rank belief: what probability does an agent assign to a representative agent having a lower return than his own?

\[ R(z) \equiv \text{Pr}(z_j \leq z|z_i = z) = \frac{\int F(\varepsilon) f(\varepsilon) g(z - \varepsilon) d\varepsilon}{\int f(\varepsilon) g(z - \varepsilon) d\varepsilon} \]

Equivalently, what is an agent’s expectation of the proportion of other agents with lower returns?
• $f$ is standard normal distribution $N(0, 1)$
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• $g$ is Student's t-distribution
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• $g$ is Student's t-distribution
  • variance of $\eta$ is unknown and distributed with inverse $\chi^2$
Rank Beliefs in the Leading Example

Figure: Rank belief function $R$. 
$R$ is differentiable and satisfies:

- **symmetry**: $R(-z) = 1 - R(z)$; in particular, $R(0) = 1/2$. 
Properties of Rank Beliefs

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- **single crossing at 1/2**: $R(z) > 1/2 > R(-z)$ whenever $z > 0$. 

Properties of Rank Beliefs

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- **symmetry**: $R(-z) = 1 - R(z)$; in particular, $R(0) = 1/2$.
- **single crossing at 1/2**: $R(z) > 1/2 > R(-z)$ whenever $z > 0$.
- **limit uniform rank beliefs**: $R(z) \to \frac{1}{2}$ as $z \to \infty$. 
Fat-Tails Assumption—Motivation

- model uncertainty:
Fat-Tails Assumption—Motivation

- model uncertainty:
  - e.g., variance uncertainty + normal $\Rightarrow$ t distribution

Empirically, changes in key economic variables have fat tails, e.g., income, prices, financial asset returns, exchange rates, GDP, ... present in many commonly used statistical models (e.g., GARCH, stochastic volatility) limit uniform rank beliefs as a primitive assumption?
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- limit uniform rank beliefs as a primitive assumption?
Figure: Rank belief function under normal idiosyncratic shocks and normal or exponential common shocks
Suppose agents follow a "cutoff" strategy, with each agent investing if his shock $z_i$ is above some critical threshold $\widehat{Z}$. 

(1) is a necessary condition for a $b\widehat{Z}$-cutoff equilibrium, also sufficient because log-concavity of $f$ implies that when an agent has a high return, she has a higher (w.r.t. FOSD) belief about other player's return.
• Suppose agents follow a "cutoff" strategy, with each agent investing if his shock $z_i$ is above some critical threshold $\hat{z}$

• an agent with shock $\hat{z}$ agent is indifferent between investing and not investing when

\[
y + \sigma \hat{z} = R(\hat{z})
\]  

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An agent with shock $\hat{Z}$ is indifferent between investing and not investing when

$$y + \sigma \hat{Z} = R(\hat{Z})$$

(1)

Following graph plots $y + \sigma \hat{Z}$ (in blue) and $R(\hat{Z})$ (in red).
Suppose agents follow a "cutoff" strategy, with each agent investing if his shock $z_i$ is above some critical threshold $\hat{z}$.

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Also, sufficient because log-concavity of $f$ implies that when an agent has a high return, she has a higher (w.r.t. FOSD) belief about other player's return.
Equilibria
• Let $z^{**}$ be largest solution to (1)
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- Corresponds to equilibrium with the least investment (invest only if $z \geq z^{**}$)

- Invest is uniquely rationalizable if and only if $z > z^{**}$
Unique Rationalizable Play

- Let $z^{**}$ be the largest solution to (1).
- Correlates to equilibrium with the least investment (invest only if $z \geq z^{**}$).
- Invest is uniquely rationalizable if and only if $z > z^{**}$.
- PROOF: Let $\bar{z}$ be the largest shock at which not invest is rationalizable and suppose $\bar{z} > z^{**}$. The payoff to investing is at least

\[
\frac{\text{own return}}{\text{proportion of others not investing}} = y + \sigma \bar{z} - R(\bar{z}) > 0,
\]

a contradiction.
Let $\bar{R}$ be the maximum possible rank belief:

$$\bar{R} = \max_{z \geq 0} R(z)$$

**Proposition**

*Invest is uniquely rationalizable whenever $x > \bar{R}$*

- equivalently, invest is uniquely rationalizable if $z > \frac{\bar{R} - y}{\sigma}$
Let $\bar{R}$ be the maximum possible rank belief:

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- for sufficiently high returns, it doesn't matter how you got there
Let $\overline{R}$ be the maximum possible rank belief:

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**Proposition**

*Invest is uniquely rationalizable whenever $x > \overline{R}$*

- equivalently, invest is uniquely rationalizable if $z > \frac{\overline{R}-y}{\sigma}$
- for sufficiently high returns, it doesn’t matter how you got there
- observe that $\frac{1}{2} < \overline{R} < 1$; thus this criterion is intermediate between risk dominance and dominant strategies
For each $x \in (\frac{1}{2}, \bar{R}]$, define critical shock size $\bar{z}(x)$ to be the largest shock at which the rank belief is $x$:

$$\bar{z}(x) = \max R^{-1}(x)$$

**Proposition**

*Invest is uniquely rationalizable if* $x \in (\frac{1}{2}, \bar{R}]$ *and* $z > \bar{z}(x)$
For each \( x \in \left( \frac{1}{2}, R \right] \), define critical shock size \( z(x) \) to be the largest shock at which the rank belief is \( x \):

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\bar{z}(x) = \max R^{-1}(x)
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**Proposition**

*Invest is uniquely rationalizable if \( x \in \left( \frac{1}{2}, R \right] \) and \( z > \bar{z}(x) \)*

- for intermediate returns, whether invest is uniquely rationalizable depends on whether there was a positive shock
• Invest will be uniquely rationalizable at fundamentals $x_i$ if reached via a large shock (left panel) but not if reached by a small shock (right panel)
Ex Ante Level Sufficient Condition

- Let $y$ be the critical level of fundamentals at which returns will exceed the rank belief whenever shocks are positive.

**Proposition**

*Invest is uniquely rationalizable whenever $x > \frac{1}{2}$ and $y > \bar{y}$*
Let $\bar{y}$ be the critical level of fundamentals at which returns will exceed the rank belief whenever shocks are positive.

Formally, define $\bar{y}$ to be the largest $y$ such that

$$R(z) \geq y + \sigma z$$

for some $z$.

**Proposition**

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Let \( \bar{y} \) be the critical level of fundamentals at which returns will exceed the rank belief whenever shocks are positive.

Formally, define \( \bar{y} \) to be the largest \( y \) such that

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\]

for some \( z \).

For small \( \sigma \), \( \bar{y} \approx \bar{R} \)

**Proposition**

*Invest is uniquely rationalizable whenever \( x > \frac{1}{2} \) and \( y > \bar{y} \)*
• For small $\sigma$, sufficient conditions are also necessary....

Proposition

If $R$ is single peaked and $y \leq \overline{R} - \sigma \overline{z}(\overline{R}) \leq \overline{y}$, invest is uniquely rationalizable only if (i) $x > \overline{R}$ or (ii) $x > \frac{1}{2}$ and $z > \overline{z}(x)$
• For small $\sigma$, sufficient conditions are also necessary.

• We get a partial converse under two additional restrictions:

**Proposition**

*If $R$ is single peaked and $y \leq \overline{R} - \sigma \overline{z}(\overline{R}) \leq \overline{y}$, invest is uniquely rationalizable only if (i) $x > \overline{R}$ or (ii) $x > \frac{1}{2}$ and $z > \overline{z}(x)$.*
Call $\theta = y + \sigma \eta$ the fundamental state; fundamental state is the population mean return and also the median agent’s return.

**Proposition**

*Invest is uniquely rationalizable for the majority if it is risk dominant ($\theta > \frac{1}{2}$) and, in addition, (i) the realized fundamentals are sufficiently high ($\theta > \overline{R}$), or (ii) the expected fundamentals were sufficiently high ($y > \overline{y}$), or (iii) the shock is sufficiently high $\eta > \overline{z}(\theta)$.***
Dynamic Game

- Infinite horizon game played in every period $t = 0, 1, \ldots$
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- Enter each period with mean $y_t$
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- Draw $\theta_t = y_t + \sigma \eta_t$
• Infinite horizon game played in every period \( t = 0, 1, \ldots \)

• Enter each period with mean \( y_t \)

• Draw \( \theta_t = y_t + \sigma \eta_t \)

• Draw \( x_{it} = \theta + \sigma \varepsilon_i = y + \sigma \eta + \sigma \varepsilon_i \)
• Infinite horizon game played in every period $t = 0, 1, \ldots$
• Enter each period with mean $y_t$
• Draw $\theta_t = y_t + \sigma \eta_t$
• Draw $x_{it} = \theta + \sigma \varepsilon_i = y + \sigma \eta + \sigma \varepsilon_i$
• Play static game
Dynamic Game

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- Period $t$ play and $\theta_t$ become common knowledge
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- Let $y_{t+1} = Y(\theta_t)$ for $t = 0, 1, ...$
Dynamic Game

- Infinite horizon game played in every period $t = 0, 1, \ldots$
- Enter each period with mean $y_t$
- Draw $\theta_t = y_t + \sigma \eta_t$
- Draw $x_{it} = \theta + \sigma \varepsilon_i = y + \sigma \eta + \sigma \varepsilon_i$
- Play static game
- Period $t$ play and $\theta_t$ become common knowledge
- Let $y_{t+1} = Y(\theta_t)$ for $t = 0, 1, \ldots$
  - for example, random walk ($y_{t+1} = \theta_t$) or reversion to the mean ($y_{t+1} = \frac{1}{2} + \kappa (\theta_t - \frac{1}{2})$)
Equilibria of the Dynamic Game

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- was there majority investment in the previous period? 
- if yes, invest whenever rationalizable 
- if not, do not invest whenever rationalizable
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Proposition

Shifts to majority investment will occur whenever invest is risk dominant \((\theta_t > \frac{1}{2})\) and, in addition, (i) the realized fundamentals are sufficiently high \((\theta_t > \bar{R})\), (ii) the expected fundamentals were sufficiently high \((y_t > \bar{y})\) or the shock was sufficiently high \(\eta_t > \bar{\eta}(\theta_t)\).
Takeaways

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  • slow news release good if you want to stay in current equilibrium (and vica versa)
  • simple mechanism that can be plugged into richer models
More generally, we can identify limit rank belief

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 No role for shocks with monotone rank beliefs and \( R_\infty = 1 \) (e.g., normality)
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Global Games

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  Higher variance of public signals / common shock required for uniqueness.
- SMALL SHOCKS PROPOSITION: Under multiplicity condition, there exists \( \Delta > 0 \) such that whenever
  \[ |x - y| \leq \Delta, \] invest is uniquely rationalizable if and only if \( y > \bar{y} \).
• If a "good" equilibrium is being played, and fundamentals are on the way down, it is better to have fundamentals drift down slowly (or bad news to be released gradually)
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• If a bad equilibrium is being played, and fundamentals are heading up, it is better to have fundamentals jump up (or good news released in chunks)
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Questions:

- If going from multiplicity to multiplicity, what explains direction of shift?
- Similarly, if going from uniqueness to multiplicity (c.f., global game arguments)
- Feels like we go from multiplicity to uniqueness?