7 A theory of the onset of currency attacks

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1 Introduction

The swiftness and devastating effect of recent financial crises pose considerable challenges for economists seeking an explanation of their onset. It is easy to give a narrative of the sequence of events leading up to the crisis with the benefit of hindsight. However this falls short of an explanation, since it begs the question of why the crisis occurred at that particular moment in time. More importantly, it does not explain the absence of a crisis in apparently similar countries, or in the same country at different moments in history. The challenge is all the more acute in the light of evidence that the onset of the Asian financial crisis of 1997 was largely unanticipated by market participants, as well as by the international agencies. Radelet and Sachs (1998) note that credit-risk spreads for borrowers in the region increased only after the crisis was in full swing, and the credit-rating agencies were largely reacting to events rather than acting in advance. Nor was there much indication from international agencies or the country analysts of normally canny investment banks that a crisis of such magnitude was brewing.

Given the difficulties in coming up with a rigorous theory, it is tempting to fall back on unexplained shifts of sentiment on the part of fickle investors, or the unexplained onset of panic among creditors as an explanation. As a formal counterpart to such an approach, multiple equilibrium models of currency attacks have gained acceptance among many commentators, and such acceptance owes a great deal to the difficulty in predicting the exact timing of currency attacks, as well as the observation that they are triggered without any apparent change in the underlying fundamentals of the economy. Such models incorporate the self-fulfilling nature of the belief in an imminent speculative attack. If speculators and exposed borrowers believe that a currency will come under attack, their actions in anticipation of this precipitate the crisis itself, while if they believe that a currency is not in danger of imminent attack, their inaction spares the currency from
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attack, thereby vindicating their initial beliefs. The onset of a currency attack is thus explained in terms of a shift from one equilibrium to another.

A large and growing literature has emerged developing this theme and formalising the intuition. Obstfeld’s work (1986, 1994, 1996) has been influential in this regard, and has served to draw a line between multiple-equilibrium models of currency attacks and the earlier generation of theories which rely on a secular deterioration of fundamentals (such as Krugman, 1979 and Flood and Garber, 1984a, 1984b), and whose argument builds on insights from models of price-fixing in exhaustible goods markets (Salant and Henderson, 1978).

However, the multiple-equilibrium approach is open to the charge that it does not fully explain a currency attack, since the shift in beliefs which leads to the movement from one equilibrium to another is left unexplained. In short, there is an indeterminacy in the theory. The beliefs of the economic actors are seen as being autonomous from the economic fundamentals, and liable to unexplained coordinated shifts. Such a view not only runs counter to our theoretical scruples against indeterminacy but, more importantly, runs counter to our intuition that bad fundamentals are somehow ‘more likely’ to trigger a crisis. Indeed, a growing empirical literature has examined the relationship between the incidence of currency attacks and the underlying economic fundamentals. A satisfactory theory of the onset of crisis must explain the shift in beliefs which trigger the attack.

In this chapter, we attempt to construct such a theory of the onset of currency attacks. The theory builds on two main features.

- The actions of diverse economic actors which exacerbate a currency crisis are mutually reinforcing. For instance, a hedge fund will find it profitable to attack a currency if it can rely on borrowers with unhedged dollar liabilities to scramble to cover their positions, and thereby exacerbate the crisis. Conversely, the borrower will find it more attractive to hedge if the currency is under attack from speculators.

- Market participants have access to a large mass of information concerning the economic fundamentals, and hence are often well informed of the underlying state of the economy. However, perhaps because of the sheer volume of information, there are small disparities in the information at the disposal of each economic actor.

The first of these features is standard in multiple-equilibrium accounts, and we adopt this basic starting point. Our innovation comes with the second feature. When there are small disparities in the information of the market participants, the indeterminacy of beliefs inherent in the multiple-equilibrium story is largely removed. Instead, it is possible to track the
shifts in beliefs as we track the shifts in the economic fundamentals. This is so since uncertainty about others' beliefs now takes on a critical role, and such uncertainty often dictates a particular course of action as being the uniquely optimal one. Even vanishingly small differences in information suffice to generate such uncertainty about others' beliefs. When we consider the sheer quantity of information available to market participants – the news wire services, in-house research, leaks from official sources, as well as the press and broadcasters, exact uniformity of information is the last thing we can expect.

Indeed, the fragmentation of the media in modern times has generated the paradoxical situation in which ever-greater quantities of information is generated and disseminated, but comes at the expense of the shared knowledge of its recipients. Apart from totalitarian regimes in which there is a single source of information (or perhaps in the heyday of the BBC Home Service), the receipt of information is rarely accompanied by the knowledge that everyone else is also receiving precisely this information at that time. Even among financial markets, the foreign exchange market is especially fragmented. Its market microstructure is characterised by the decentralised nature of the trade necessitated by round-the-clock trading and the geographical spread which goes with it. At its most basic, a speculative attack is a resolution of a coordination problem among the diverse interested parties – both foreign and domestic. Small disparities of information determine the outcome of such coordination problems.

In earlier work (Morris and Shin, 1998), we have illustrated this point in the context of a simple static model where speculators observe accurate, but idiosyncratic, signals concerning the fundamentals, and they hold uniform prior beliefs about the fundamentals. In this case, the multiplicity of equilibrium is completely eliminated, and a unique outcome emerges in equilibrium.

Here, we develop this line of inquiry further, and clarify the role of differential information in currency attacks. The static framework and the strong distributional assumptions in our earlier work did not allow us to distinguish between the issue of the uniqueness of equilibrium from the more general issue of how the set of equilibria of the imperfect information game is affected by the departures from common knowledge. Although changes in the equilibrium set to shifts in the nature of differential information is to be expected, uniqueness of equilibrium requires additional pieces of the jigsaw to be in place. Also, the static nature of the model in our earlier work detracted from the goal of serving as a theory of the onset of currency attacks. Any such theory must take into account the evolution of the fundamentals over time, and incorporate differential information in this dynamic context.
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In what follows, the fundamentals evolve according to a Brownian motion process, and market participants monitor the fundamentals accurately, but with small differences in their information. We demonstrate the existence of an accompanying stochastic process — called the ‘hurdle process’ — whereby, as long as the fundamentals lie above the realisation of the hurdle process, there is no currency attack. However, as soon as the fundamental process falls below the hurdle process, an attack inevitably follows. The imagery is intended to be suggestive. As long as fundamentals can negotiate the hurdle, there is no attack. However, as soon as it ‘trips over’ the hurdle, an attack is triggered. This hurdle process also has the feature that it moves in the opposite direction to the fundamentals. Thus, when fundamentals deteriorate, the hurdle shifts upwards, making it more difficult to clear the hurdle.

We readily acknowledge that such a model is still too rudimentary to yield detailed policy implications. However, it makes a small step in the direction of giving us the framework in which such questions can be addressed within the theory, rather than appealing to forces outside it.

2 Elements of a theory

Defending a currency peg in adverse circumstances entails large costs for the government or monetary authorities. The costs bear many depressingly familiar symptoms — collapsing asset values, rising bankruptcies, the loss of foreign exchange reserves, high interest rates and the resulting reduction in demand leading to increases in unemployment and slower growth. Whatever the perceived benefits of maintaining a currency peg, and whatever their official pronouncements, all monetary authorities have a pain threshold at which the costs of defending the peg outweighs the benefits of doing so. Understanding the source and the severity of this pain is a key to understanding the onset of currency attacks.

Facing the monetary authority is an array of diverse private sector actors, both domestic and foreign, whose interests are affected by the actions of the other members of this group, and by the actions of the monetary authority. The main actors are domestic corporations, domestic banks and their depositors, foreign creditor banks, and outright speculators — whether in the form of hedge funds or the proprietary trading desks of the international financial houses. Two features stand out, and deserve emphasis:

- Each actor faces a choice between actions which exacerbate the pain of maintaining the peg and actions which are more benign.
• The more prevalent are the actions which increase the pain of holding the peg, the greater is the incentive for an individual actor to adopt the action which increases the pain. In other words, the actions which tend to undermine the currency peg are mutually reinforcing.

For domestic corporations with unhedged foreign currency liabilities, they can either attempt to hedge their positions or not. The action to hedge their exposure – of selling baht to buy dollars in forward contracts, for example – is identical in its mechanics (if not in its intention) to the action of a hedge fund which takes a net short position in baht. For domestic banks and finance houses which have facilitated such dollar loans to local firms, they can either attempt to hedge their dollar exposure on their balance sheets or not. Again, the former action is identical in its consequence to a hedge fund short-selling baht. As a greater proportion of these actors adopt the action of selling the domestic currency, the greater is the pain to the monetary authorities, and hence the greater is the likelihood of abandonment of the peg. This increases the attractiveness of selling baht. In this sense, the actions which undermine the currency peg are mutually reinforcing. They are 'strategic complements', in the sense used in game theory.

Indeed, the strategic effects run deeper. As domestic firms with dollar liabilities experience difficulties in servicing their debt, the banks which have facilitated such dollar loans attempt to cover their foreign currency losses and improve their balance sheet by a contraction of credit. This in turn is accompanied by a rise in interest rates, fall in profit and a further increase in corporate distress. For foreign creditor banks with short-term exposure, this is normally a cue to cut off credit lines, or to refuse to roll over short-term debt. Even for firms with no foreign currency exposure, the general contraction of credit increases corporate distress. Such deterioration in the domestic economic environment exacerbates the pain of maintaining the peg, thereby serving to reinforce the actions which tend to undermine it. To make matters worse still, the belated hedging activity by banks is usually accompanied by a run on their deposits, as depositors scramble to withdraw their money.

Table 7.1 contains a (somewhat simplistic) taxonomy of the various actors and their actions which undermine the peg. The feature to be emphasized is the increased pain of maintaining the peg in the face of widespread adoption of such actions, and hence the mutually reinforcing nature of the action which undermines the peg. The greater is the prevalence of such actions, the more attractive such actions become to the individual actor.

To be sure, the actual motives behind these actions are as diverse as the actors themselves. A currency speculator rubbing his hands and looking on in glee as his target country descends into economic chaos has very different
Table 7.1 Undermining the peg

<table>
<thead>
<tr>
<th>Actor</th>
<th>Action(s) undermining peg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speculators</td>
<td>Short sell Baht</td>
</tr>
<tr>
<td>Domestic firms</td>
<td>Sell Baht for hedging purposes</td>
</tr>
<tr>
<td>Domestic banks</td>
<td>Sell Baht for hedging purposes</td>
</tr>
<tr>
<td>Foreign banks</td>
<td>Reduce credit to domestic firms</td>
</tr>
<tr>
<td>Depositors</td>
<td>Refuse to roll over debt</td>
</tr>
<tr>
<td></td>
<td>Withdraw deposits</td>
</tr>
</tbody>
</table>

motives from a desperate owner of a firm in that country trying frantically to salvage what he can, or a depositor queueing to salvage her meagre life savings. However, whatever the motives underlying these actions, they are similar in their consequences. They all lead to greater pains of holding to the peg, and hence hasten its demise.

For the purposes of the formal development of the theory, we will abstract from the diverse motives of the private sector actors, and simply treat everyone as being a potential 'speculator' against the currency. Hence, in what follows, the label of 'speculator' should be taken to apply to the array of economic actors discussed above.

We summarise by \( \theta \) the overall perception of the monetary authorities concerning the robustness of the underlying economy and, by implication, the ease with which the monetary authorities can withstand speculative selling of the currency. When \( \theta \) is low, the economy is in bad shape and the costs of defending the currency peg is high. When \( \theta \) is high, the reverse is true and the cost of defending the peg is low. When \( \theta \) is sufficiently low, the monetary authorities abandon the peg irrespective of the actions of the speculators. Conversely, when \( \theta \) is sufficiently high, the government maintains the peg irrespective of the actions of speculators. However, the cost of defending the peg depends on the extent to which the peg comes under concerted attack. For intermediate values of \( \theta \), the cost of maintaining the peg is pivotal in the government's decision on whether to abandon it.

Let \( a(\theta) \) be the degree of ferocity of the attack on the currency which is just sufficient to induce the monetary authorities to abandon the peg, as measured by the proportion of speculators who sell the currency (we assume that each speculator has the binary choice of whether to attack the currency, or not to do so). In other words, if proportion \( a(\theta) \) or greater attack the currency at state \( \theta \), the monetary authorities abandon the peg, while if the proportion attacking the currency is less than \( a(\theta) \), the government maintains the peg. We further assume that

- There is \( \theta \) such that \( a(\theta) = 0 \) for \( \theta \leq \theta \)
• There is $\bar{\theta}$ such that $a(\theta)$ is undefined for $\theta \leq \bar{\theta}$

• $a(\theta)$ is strictly increasing in $\theta$ when $0 < a(\theta) < 1$, and there is a bound $b$ on the slope of $a(\cdot)$, so that $0 < b \leq a'(\theta)$.

2.1 Evolution of $\theta$

Time is discrete, and advances in increments of $\Delta > 0$. The value of $\theta$ at time $t$ is denoted by $\theta(t)$. Conditional on $\theta(t)$, the value of $\theta$ at time $t + \Delta$ is distributed normally with mean $\theta(t)$ and variance $\Delta$. Such a feature would result if observations of $\theta$ are snapshots of a process which evolved according to the Brownian motion process

$$d\theta = z\sqrt{\Delta}dt$$ (7.1)

where $z$ is the standard normal random variable.

The monetary authorities observe $\theta$ perfectly (after all, $\theta$ is the perception of the monetary authorities). However, other parties do not observe $\theta$ perfectly. In particular, the speculators are able to observe $\theta$ only after a delay of $\Delta$. Thus, at time $t + \Delta$, they observe $\theta(t)$.

However, although the speculators do not observe the current value of $\theta$, they do have a noisy signal of the current $\theta$. Speculator $i$ observes at time $t$ the random variable

$$x_i(t) = \theta(t) + \eta_i$$ (7.2)

where $\eta_i$ is a normal random variable with mean zero, and variance $\epsilon \Delta$, where $\epsilon$ is a small positive number. So, the variance of the noise term is $\epsilon$ times the one-period-ahead variance of $\theta$ itself. Furthermore, each $\eta_i$ is independent of $\theta$, and of $\eta_j$ for all $j \neq i$.

To summarise, at time $t$, the information at the disposal of the monetary authorities and the speculators are as follows.

• Monetary authorities: $\{\theta(t)\}$
• Speculator $i$: $\{\theta(t - \Delta), x_i(t)\}$.

2.2 Payoffs

In each period, the speculators decide whether to attack the currency or not, based on their information. There is a cost of attacking the currency,
Table 7.2 Matrix of payoffs

<table>
<thead>
<tr>
<th></th>
<th>Peg maintained</th>
<th>Peg abandoned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attack</td>
<td>$-c$</td>
<td>$1-c$</td>
</tr>
<tr>
<td>Refrain</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

given by a constant $c > 0$. As well as the transaction costs associated with attacking a currency, $c$ incorporates any differences in the interest rates between the target currency and the dollar. For a speculator who borrows the target currency and sells it for dollars, the higher interest cost of the borrowing can sometimes be substantial. Our model does not address the determination of this cost. We assume it to be a known parameter.

The monetary authorities observe the aggregate short-selling of the speculators and maintains the peg at $\theta$ if and only if the proportion of speculators who attack the currency does not exceed the threshold level $a(\theta)$. When the currency peg is removed, the currency depreciates by a known amount $D > 0$, and remains at this lower level forever. We normalise payoffs and assume from now on that $D = 1$. We thus have the matrix of payoffs to a particular speculator shown in table 7.2.

If $\theta$ were common knowledge among the speculators, there is the familiar multiplicity of equilibria in the range $(\theta, \bar{\theta})$. If speculators believe that the peg will be maintained, they refrain from attacking the currency, which leads to the peg being maintained. If, however, they believe that the peg will be abandoned, they attack the currency, leading to its downfall.

However, common knowledge of fundamentals would be an inappropriate assumption in the context of financial markets, as we shall argue below.

2.3 Joint distributions

In thinking about the joint distributions generated by our model, recall that if $(X, Y)$ has a bivariate normal distribution, then the conditional distribution of $X$ given $Y = y$ is normal with mean

$$\mu_x + \left(\rho \sigma_x / \sigma_y\right)(y - \mu_y)$$

and variance

$$\sigma_x^2 (1 - \rho^2)$$

where $\mu$ denotes the mean of the subscripted random variable, $\sigma^2$ denotes its variance and $\rho$ is the correlation coefficient between $X$ and $Y$. 
In our case, we will be interested in the one-step-ahead covariances conditional on \( \theta \) at time \( t - \Delta \). From our assumptions,
\[
\text{cov} \left( x_i(t), x_j(t) \mid \theta(t - \Delta) \right) = \text{cov} \left( x_i(t), \theta(t) \mid \theta(t - \Delta) \right) \\
= \text{var} \left( \theta(t) \mid \theta(t - \Delta) \right) \\
= \Delta.
\]

Similarly,
\[
\text{var} \left( x_i(t) \mid \theta(t - \Delta) \right) = \text{var} \left( \theta(t) \mid \theta(t - \Delta) \right) + \text{var} \left( \eta_i \right) \\
= \Delta \left( 1 + \epsilon \right).
\]

When no confusion is possible, we will economise on notation, and drop the time argument. Unless otherwise stated, all covariances are conditional on the realisation of \( \theta \) in the previous period. Hence, we write \( \text{cov}(x_i, x_j) \) and \( \text{var}(x_i) \) for the expressions above.

The one-step-ahead correlation coefficients between \( x_i \) and \( x_j \) and between \( x_i \) and \( \theta \) are given by
\[
\rho(x_i, x_j) = \frac{\text{cov}(x_i, x_j)}{\sqrt{\text{var}(x_i) \text{var}(x_j)}} \\
= \frac{\text{var}(\theta)}{\text{var}(\theta) + \text{var}(\eta_i)} \\
= \frac{1}{1 + \epsilon}
\]

and
\[
\rho(x_i, \theta) = \frac{\text{cov}(x_i, \theta)}{\sqrt{\text{var}(x_i) \text{var}(\theta)}} \\
= \frac{\text{var}(\theta)}{\sqrt{\text{var}(\theta) + \text{var}(\eta_i)} \text{var}(\theta)} \\
= \frac{\Delta}{\sqrt{\Delta(1 + \epsilon)\Delta}} \\
= \frac{1}{\sqrt{1 + \epsilon}}.
\]

In both cases, the correlation is high when \( \epsilon \) is small, and the speculators have good information concerning \( \theta \) and the signals of others. In terms of the inference problem, the distributions of interest are, first, the conditional distribution of \( \theta(t) \) on \( x_i(t) \) and the previous realisation of \( \theta \) denoted by
\[
f(\theta \mid x_i, \theta_{t-\Delta}) \quad (7.3)
\]
and the conditional distribution of \( x_i(t) \) on \( \theta(t) \), denoted by \( f(x_i | \theta) \). The former summarises the beliefs of speculator \( i \) concerning the fundamentals, while the latter gives the distribution of signals for a given state of fundamentals. We know that

\[
f(x_i | \theta) = \eta_i
\]

so that it is normal with mean zero and variance \( \epsilon \Delta \). As for \( f(\theta | x_i, \theta_{-\Delta}) \), normality of the underlying random variables implies that \( f(\theta | x_i, \theta_{-\Delta}) \) is also normal whose mean is given by

\[
E(\theta) + \frac{\text{cov}(x_i, \theta)}{\text{var}(x_i)} (x_i - E(x_i))
\]

\[
= \theta_{-\Delta} + \frac{\text{var}(\theta)}{\text{var}(\theta) + \text{var}(x_i)} (x_i - \theta_{-\Delta})
\]

\[
= \left( \frac{\epsilon}{1 + \epsilon} \right) \theta_{-\Delta} + \left( \frac{1}{1 + \epsilon} \right) x_i.
\]

In other words, when trader \( i \) observes signal \( x_i \), she forms her beliefs on the current value of \( \theta \) by taking a convex combination of her current signal \( x_i \) and the previous realisation of \( \theta \). As the signal gets more accurate (i.e. as \( \epsilon \) becomes small), the trader puts more weight on her signal, and less on the prior realisation. The variance of \( f(\theta | x_i) \) is given by \( \text{var}(\theta)(1 - \rho^2) \), or

\[
\frac{\epsilon \Delta}{1 + \epsilon}
\]

To summarise,

- \( f(x_i | \theta) \) is normal with mean zero and variance \( \epsilon \Delta \).
- \( f(\theta | x_i, \theta_{-\Delta}) \) is normal with mean \( \left( \frac{\epsilon}{1 + \epsilon} \right) \theta_{-\Delta} + \left( \frac{1}{1 + \epsilon} \right) x_i \) and variance \( \frac{\epsilon \Delta}{1 + \epsilon} \).
- The correlation between \( x_i \) and \( x_j \) is \( 1/(1 + \epsilon) \).

2.4 Failure of common belief

Although the signals of the speculators are highly correlated when \( \epsilon \) is small, there is a qualitative difference between the case when \( \epsilon \) is small but positive and when \( \epsilon \) is precisely zero for the degree of common knowledge. In the former, there is common knowledge of the fundamentals, but in the latter even approximate common knowledge fails, as we shall demonstrate.
The fact that an individual believes some feature of the economy to be this or that way is as much a description of the world as any statement about the fundamentals of the economy. For event $E$, we can associate those states of the world at which some group of individuals hold certain beliefs concerning $E$. Define the operator $B_q(.)$ as:

$$B_q(E) = \left\{ \theta \mid \text{proportion } q \text{ or higher of speculators} \right.$$  

believe $E$ with probability $q$ or higher $\}$.

When $\theta$ belongs to $B_q(E)$, proportion $q$ or higher of speculators believe event $E$ with probability $q$ or higher at $\theta$. Consider the event $E = [\theta, \infty)$ — i.e. the event that the fundamentals are consistent with the peg. When $\epsilon = 0$, we have $B_1(E) = E$, and hence

$$E = B_1(E) = B_1(B_1(E)) = B_1(B_1(B_1(E))) = \ldots$$

for any number of iterations of the operator $B_1(.)$, so that whenever fundamentals are consistent with the peg (i.e. when $\theta \in E$), everyone believes this with probability 1, everyone believes that everyone believes it, everyone believes that everyone believes it, and so on, without bound.

Contrast this with the case when $\epsilon$ is small, but positive. Consider when at least 90 per cent or speculators believe $E$ with probability at least 0.9 (see figure 7.1). Graph 7.1a illustrates the density $f(\theta \mid x, \theta_\Delta)$ — the posterior density over $\theta$ given the information $(x, \theta_\Delta)$. In order for a speculator to place belief 0.9 or greater on the event $E$, the signal $x$ must be at least as high as $x_*$. Graph 7.1b illustrates the density of the signals generated by the noise. In order for 90 per cent of speculators to receive a signal greater than $x_*$, the value of $\theta$ must be at least $\theta_*$. Thus, the event in which at least 90 per cent of speculators believe $E$ with probability at least 0.9 is given by the interval $[\theta_*, \infty)$. In other words,

$$B_{0.9}(E) = \left[ \theta_*, \infty \right) \supset \ E.$$

Indeed, for $q > 1/2$, we have

$$\theta \in B_q(\ldots (B_q(B_q(E))) \ldots)$$

for some finite number of iterations. In other words, when $\epsilon$ is small but positive, even approximate common knowledge of fundamentals fails.

We should think of common belief not in terms of the mental gymnastics of higher-order beliefs, but in terms of the ‘transparency’ of the situation. When two individuals are seated across the same table in a well-lit room, we can reasonably claim that there is common knowledge of this fact, given its transparency to both individuals. The fundamentals, in this case,
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satisfy a fixed-point property in that this situation obtains if both individuals know that this is so.³

Away from such special circumstances, common knowledge, and even approximate common knowledge is very rarely in place in the real world. For financial markets, common knowledge of fundamentals is a singularly inappropriate assumption. In what follows, therefore, the results which stand in contrast to the benchmark case should be attributed to the failure of common knowledge. We build on the work of game theorists who have
investigated the effects of higher order uncertainty (Rubinstein, 1989; Monderer and Samet, 1989; Carlsson and Van Damme, 1993a, 1993b; Morris, Rob and Shin, 1995; Kajii and Morris, 1997). Morris and Shin (1997) is a survey of some of the main results to date.

3 The incomplete information game

At date \( t \), if the currency peg is still intact, each speculator decides whether or not to attack the currency based on her information. A strategy for speculator \( i \) is a function

\[
(x, \theta_i) \rightarrow \{\text{Attack, Refrain}\}.
\]  

(7.5)

In principle, a speculator can choose an action conditional on the whole history of \( \theta \), but since history reveals no more information than the previous realisation, we restrict attention to Markov strategies of the above form.

The monetary authority observes \( \theta \) and the proportion, \( s \), of speculators who attack. It abandons the peg if and only if

\[
s \geq a(\theta).
\]

If the peg is abandoned, it is never reinstated. In effect, the game ends when the peg is abandoned. The payoffs of the game are given in table 7.2.

We can now state the main result of the chapter. We shall do it in terms of the following pair of theorems.4

**Theorem 1** For \( \varepsilon \) sufficiently small, there is a stochastic process \( \{h(t)\} \) such that the currency peg is maintained as long as \( \theta > h \), but the peg is abandoned as soon as \( \theta \leq h \).

**Theorem 2** \( h(t) \geq h(t - \Delta) \) if \( \theta(t - \Delta) \leq \theta(t - 2\Delta) \).

Theorem 1 states that when market participants have sufficiently accurate information concerning the fundamentals, we can construct a stochastic process – an accompanying ‘hurdle process’ – such that the onset of a speculative attack can be characterised in terms of the fundamentals ‘tripping over’ the hurdle.

Theorem 2 states that the hurdle moves in the opposite direction to the fundamentals. So, when economic fundamentals deteriorate, the hurdle becomes higher than before. Conversely, when fundamentals improve, the
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hurdle falls further away. This implies that when the fundamentals deteriorate, the prospect of a currency attack increases more than proportionately to the deterioration of the fundamentals.

The model and conclusions presented here may be contrasted with our earlier work, Morris and Shin (1998). The economic model in this chapter is more reduced-form; but it captures the same essential features. Our earlier work made distributional assumptions that guaranteed uniqueness of equilibrium; the normal processes assumed in this chapter do not guarantee uniqueness, but uniqueness is guaranteed for sufficiently small noise and, as we discuss below, this noise does not need to be too small. Finally, by explicitly modelling a dynamic process, we are able to see how the hurdle changes through time. Previous period fundamentals influence the threshold at which an attack occurs because they influence speculators' beliefs about other speculators' beliefs.

3.1 Overview of the argument

Before giving the detailed proofs for theorems 1 and 2, we give an outline of the shape of our argument. The first step in our analysis is to identify bounds on equilibrium actions. For any given profile of strategies by the speculators, denote by

$$\pi(x, \theta_{-\Delta})$$

the proportion of traders who attack the currency given \((x, \theta_{-\Delta})\).

Define the set \(\mathcal{E}\) as the set of all \(\pi\) which may arise in an equilibrium of the game. In other words, \(\pi \in \mathcal{E}\) if and only if there is some equilibrium in which the proportion of speculators who attack given \((x, \theta_{-\Delta})\) is given by \(\pi\) \((x, \theta_{-\Delta})\). Define

$$\underline{\pi}(\theta_{-\Delta}) = \inf \{x \mid \pi(x, \theta_{-\Delta}) < 1 \text{ and } \pi \in \mathcal{E}\}$$

$$\overline{\pi}(\theta_{-\Delta}) = \inf \{x \mid \pi(x, \theta_{-\Delta}) > 0 \text{ and } \pi \in \mathcal{E}\}.$$  

Thus, \(\underline{\pi}(\theta_{-\Delta})\) is the greatest lower bound on the signal at which at least some of the traders do not attack the currency. Thus, if \(x < \underline{\pi}(\theta_{-\Delta})\), we can be sure that every speculator attacks given \((x, \theta_{-\Delta})\) in every equilibrium, while if \(x > \overline{\pi}(\theta_{-\Delta})\), every speculator refrains given \((x, \theta_{-\Delta})\) in every equilibrium.

We will show that the bounds \(\underline{\pi}(\theta_{-\Delta})\) and \(\overline{\pi}(\theta_{-\Delta})\) can be identified by constructing a continuous function \(U(x, \theta_{-\Delta})\) which has four features.

* \(U \rightarrow 1 - c\) as \(x \rightarrow -\infty\), and \(U \rightarrow -c\) as \(x \rightarrow -\infty\)
• $\gamma(\theta_{-\Delta}) = \min \{ x \mid U(x, \theta_{-\Delta}) = 0 \}$
• $\bar{x}(\theta_{-\Delta}) = \max \{ x \mid U(x, \theta_{-\Delta}) = 0 \}$
• $U(x, \theta_{-\Delta})$ is decreasing in $\theta_{-\Delta}$.

The function $U$ is positive for small values of $x$, and is negative for large values. It is continuous, and so must cut the horizontal axis at least once. The smallest value of $x$ at which $U$ cuts the horizontal axis is shown to be $\gamma(\theta_{-\Delta})$, while the largest value at which $U$ cuts the horizontal axis is shown to be $\bar{x}(\theta_{-\Delta})$. Thus, once this function has been identified, the characterisation of equilibrium actions can be reduced to the simple task of checking where it cuts the horizontal axis.

Moreover, this function is also shown to have the feature that, for sufficiently small $\epsilon$,

$$\frac{\partial U}{\partial x} < 0,$$

so that $U$ cuts the horizontal axis precisely once. This implies that $\gamma(\theta_{-\Delta}) = \bar{x}(\theta_{-\Delta})$, so that we can tie down equilibrium actions precisely. For sufficiently small $\epsilon$, there is a unique $\theta'$ for each $\theta_{-\Delta}$ such that, in any equilibrium, the government abandons the currency peg if

$$\theta \leq \theta'.$$  \hspace{1cm} (7.10)

The hurdle process $\{h(t)\}$ can then be constructed by defining the realisation of $h(.)$ at time $t$ to be the value of $\theta'$ associated with the realisation $\theta(t - \Delta)$. Theorem 1 follows from this definition and (7.10).

We now present the proofs of theorems 1 and 2, as well as illustrating the argument with a number of simulations.

4 The argument

4.1 Conditional expected payoff

Denote by $s(\theta, \pi)$ the proportion of speculators who end up attacking the currency when the state of fundamentals is $\theta$, given the aggregate selling strategy $\pi$. It is given by

$$s(\theta, \pi) = \int_{-\infty}^{\infty} \pi(x, \theta_{-\Delta}) f(x \mid \theta) dx.$$  \hspace{1cm} (7.11)
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Denote by \( A(\theta_{-\Delta}, \pi) \) the event in which the monetary authority abandons the currency peg when the speculators’ aggregate short selling is \( \pi \). In other words,

\[
A(\theta_{-\Delta}, \pi) = \{ \theta \mid s(\theta, \pi) \geq u(\theta) \}. \tag{7.12}
\]

When the government abandons the peg, there is a devaluation in the currency of \( D = 1 \). Since a speculator does not observe \( \theta \) directly, the optimal decision rests on the expected payoff from attacking the currency conditional on the signal \( x \) received. We denote by \( u(x, \theta_{-\Delta}, \pi) \) the expected payoff from attacking the currency conditional on \( (x, \theta_{-\Delta}) \) when the aggregate short selling is given by \( \pi \). Then,

\[
u(x, \theta_{-\Delta}, \pi) = \int_{A(\theta_{-\Delta}, \pi)} f(\theta \mid x, \theta_{-\Delta}) d\theta - c. \tag{7.13}\]

4.2 Defining \( U(x, \theta_{-\Delta}) \)

The function \( U(x, \theta_{-\Delta}) \) is defined to be the expected payoff conditional on \( (x, \theta_{-\Delta}) \) when speculators follow the strategy of attacking the currency if the realisation of the signal is \( x \) or lower. In other words,

\[
U(x, \theta_{-\Delta}) = u(x, \theta_{-\Delta}, I_x)
= \int_{A(\theta_{-\Delta}, I_x)} f(\theta \mid x) d\theta - c \tag{7.14}
\]

where \( I_x(y) \) is the indicator function which takes the value 1 when \( y \leq x \), and takes value 0 when \( y > x \). That is

\[
I_x(y) = \begin{cases} 
1 & \text{if } y \leq x \\
0 & \text{if } y > x.
\end{cases} \tag{7.15}
\]

In order to express \( U \) more succinctly, we characterise the event \( A(\theta_{-\Delta}, I_x) \) – i.e. the event in which the peg is abandoned when the speculators’ aggregate sales of the currency given by \( I_x \).

The distribution of \( x \) given \( \theta \) is normal with mean \( \theta \) and standard deviation \( \sqrt{\epsilon \Delta} \). Denoting by \( \Phi(\kappa, \mu, \sigma) \) the cumulative normal distribution at \( k \) when the mean is \( \mu \) and the standard deviation is \( \sigma \), we have

\[
s(\theta, I_x) = \Phi\left(x, \theta, \sqrt{\epsilon \Delta}\right)
= 1 - \Phi\left(\theta, x, \sqrt{\epsilon \Delta}\right).
\]
So, \( A(\pi_\theta, \theta_{-\Delta}) = (-\infty, \psi(x)) \) is the unique \( \theta \) which solves
\[
1 - \Phi\left( \theta, x, \sqrt{\epsilon \Delta} \right) = a(\theta).
\] (7.16)

The solution is unique, since \( a \) is increasing, while \( 1 - \Phi\left( \theta, x, \sqrt{\epsilon \Delta} \right) \) is decreasing. Figure 7.2 illustrates \( \psi(x) \)

Hence,
\[
U(x, \theta_{-\Delta}) = \int_{-\infty}^{\psi(x)} f(\theta | x, \theta_{-\Delta}) \, d\theta - c
\]
\[
= \Phi\left( \psi(x), \frac{x + \epsilon \theta_{-\Delta}}{1 + \epsilon}, \sqrt{\frac{\epsilon \Delta}{1 + \epsilon}} \right) - c.
\] (7.17)

We note the following properties of this function:

* \( \theta < \psi(x) < \bar{\theta} \)
* \( U \) is positive for small \( x \) (tends to 1 - \( c \))
* \( U \) negative for large \( x \) (tends to \( -c \))
* \( U \) is continuous in \( x \).

We can conclude therefore, that \( U \) is positive for small values of \( x \), negative for large values of \( x \), and that it crosses the horizontal axis at least once. Consider the smallest and largest values of \( x \) for which \( U = 0 \). We can prove:

**Lemma 1**

\[
\bar{x}(\theta_{-\Delta}) = \min \left\{ x \mid U(x, \theta_{-\Delta}) = 0 \right\}
\]
\[
\bar{x}(\theta_{-\Delta}) = \max \left\{ x \mid U(x, \theta_{-\Delta}) = 0 \right\}.
\]

In our argument for this result, we will need the following preliminary result.

**Lemma 2** If \( \pi \succeq \pi' \), then \( u(x, \theta_{-\Delta}, \pi) \succeq u(x, \theta_{-\Delta}, \pi') \).

Lemma 2 states that the payoff to attacking the currency is higher when the attack on the currency is stronger. In other words, speculators' decisions to attack are strategic complements. To prove lemma 2, note that if \( \pi(x, \theta_{-\Delta}) \succeq \pi'(x, \theta_{-\Delta}) \), we have \( s(\theta, \pi) \succeq s(\theta, \pi') \) for every \( \theta \), so that
\[
A(\pi, \theta_{-\Delta}) \succeq A(\pi', \theta_{-\Delta}).
\]
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Figure 7.2  \( \psi(x) \)

\[
\begin{align*}
\psi(x) &= f(\theta|x) \, d\theta - c \\
&\geq \int_{A(\theta_{\Delta}, \pi)} f(\theta|x) \, d\theta - c \\
&= u(x, \theta_{\Delta}, \pi).
\end{align*}
\]

In other words, the event in which the currency peg is abandoned is strictly larger under \( \pi \). Thus,

\[
\begin{align*}
\chi(x, \theta_{\Delta}, \pi) &= \inf \{x \mid 0 < \pi(x, \theta_{\Delta}) < 1 \text{ and } \pi \in \mathcal{E}\} \\
&\leq \sup \{x \mid 0 < \pi(x, \theta_{\Delta}) < 1 \text{ and } \pi \in \mathcal{E}\} \\
&\leq \chi(\theta_{\Delta}).
\end{align*}
\]

Now, if \( \pi < 1 \), then some speculators do not attack. This is consistent with equilibrium only if the payoff from not attacking is at least as high as attacking. By continuity, the same is true at \( \chi(\theta_{\Delta}) \). Hence,

\[
\chi(\theta_{\Delta}) = 0.
\]

By strategic complementarity of actions (lemma 2),

\[
U(\chi(\theta_{\Delta}), \theta_{\Delta}) \leq u(\chi(\theta_{\Delta}), \theta_{\Delta}, \pi) \leq 0
\]

implying that

\[
\min \{x \mid U(x, \theta_{\Delta}) = 0\} \leq \chi(\theta_{\Delta}).
\]

Meanwhile, we can construct a symmetric equilibrium in switching strategies at \( \min \{x \mid U(x, \theta_{\Delta}) = 0\} \). In other words, there is an equilibrium in which every speculator attacks if and only if

\[
x \leq \min \{x \mid U(x, \theta_{\Delta}) = 0\}.
\]
To see this, suppose that every speculator follows this strategy. Then, by construction, the speculator who receives the marginal message \( \min \{ x \mid U(x, \theta_{-\Delta}) = 0 \} \) is indifferent between attacking the currency and not. But from (7.17) the expected payoff from attacking the currency is decreasing in the message \( x \). Thus, any speculator who receives a message greater than the marginal one prefers to refrain, while a speculator who receives a message lower than the marginal one prefers to attack. Thus, we have an equilibrium. The fact that there is a symmetric equilibrium in switching strategies at \( \min \{ x \mid U(x, \theta_{-\Delta}) = 0 \} \) implies
\[
\min \{ x \mid U(x, \theta_{-\Delta}) = 0 \} \geq \bar{z}(\theta_{-\Delta}),
\] (7.21)
so that together with (7.20),
\[
\min \{ x \mid U(x, \theta_{-\Delta}) = 0 \} = \bar{z}(\theta_{-\Delta}).
\] (7.22)
There is an analogous argument for
\[
\min \{ x \mid U(x, \theta_{-\Delta}) = 0 \} = \bar{\bar{z}}(\theta_{-\Delta}).
\] (7.23)
This completes the proof of lemma 1.

The shape of the \( U \) function determines the equilibrium set, and when \( \epsilon \) is small, \( U \) is a monotonic function of \( x \).

**Lemma 3** \( \partial U / \partial x < 0 \) if \( \epsilon \) is sufficiently small.

**Proof** From
\[
s(\psi(x), I_{\epsilon}) = \Phi \left( x, \psi(x), \sqrt{\epsilon \Delta} \right),
\]
and denoting by \( \Phi_n \) the partial derivative of \( \Phi \) with respect to its \( n \)th argument, total differentiation with respect to \( x \) yields
\[
\Phi_1 + \Phi_2 \psi'(x) = a'(\psi(x)) \psi'(x).
\]
Rearranging,
\[
\psi'(x) = \frac{\Phi_1}{a'(\psi(x)) - \Phi_2}.
\]
However, \( \Phi_1 \) is the value of the normal density at \( x \), so that \( \Phi_1 = \phi \left( x, \psi(x), \sqrt{\epsilon \Delta} \right) \), where \( \phi \) is the density corresponding to \( \Phi \). The partial derivative \( \Phi_2 \) is the negative of \( \Phi_1 \). Thus,
\[
\psi'(x) = \frac{\phi(x, \psi(x), \sqrt{\epsilon \Delta})}{a'(\psi(x)) + \phi(x, \psi(x), \sqrt{\epsilon \Delta})}.
\] (7.24)
Consider \( \Phi(k, \mu, \sigma) \). If both \( k \) and \( \mu \) are differentiable functions of \( x \) while the variance is constant, then \( \Phi(k, \mu, \sigma) \) is decreasing in \( x \) if \( k' < \mu' \). Hence,

\[
\frac{\partial U}{\partial x} < 0 \Leftrightarrow \psi'(x) < \frac{1}{1 + \epsilon}.
\]

(7.25)

We know that

\[
\psi'(x) = \frac{\phi(x, \psi(x), \epsilon \Delta)}{a'(\psi(x)) + \phi(x, \psi(x), \epsilon \Delta)}
\]

\[
= \frac{1}{\frac{\psi}{\psi} + 1'}
\]

Thus, \( \psi'(x) < \frac{1}{1 + \epsilon} \) if \( a'/\phi > \epsilon \), or

\[
a' > \epsilon \phi.
\]

(7.26)

Any normal density attains its maximum value at its mean and this is

\[
\frac{1}{\sigma \sqrt{2\pi}}
\]

where \( \sigma \) is its standard deviation and \( \pi \) is the number pi (not to be confused with the use we have made of it so far). In our case, \( \sigma = \sqrt{\epsilon \Delta} \). Thus,

\[
\epsilon \phi \geq \frac{\epsilon}{2\Delta \pi}
\]

(7.27)

Hence \( \epsilon \phi \to 0 \) as \( \epsilon \to 0 \). Thus, for sufficiently small \( \epsilon \) the inequality (7.26) holds. This is sufficient for \( U \) to be decreasing in \( x \). This proves lemma 3.

Figure 7.3 illustrates the point.

The horizontal axis measures \( x \), the vertical axis measures \( \theta \). For any given realisation of \( \theta \) in the previous period, the mean of the conditional distribution \( f(\theta | x, \theta_{-\Delta}) \) is a linear function of \( x \), with slope \( 1/(1 + \epsilon) \). The smaller is \( \epsilon \), the greater is the weight placed on the noisy signal and smaller is the weight placed on the previous realisation of \( \theta \). The whole distribution \( f(\theta | x, \theta_{-\Delta}) \) is depicted above. The function \( \psi() \) maps \( x \) into the value of \( \theta \) at which the government switches from maintaining the peg to abandoning the peg. \( \psi(x) \) takes values in the open interval \((\theta, \theta)\), and is increasing.

From the diagram, we can see that \( U(x, \theta_{-\Delta}) + c \) is given by the area under \( f(\theta | x, \theta_{-\Delta}) \) to the left of the point \( \psi(x) \). This is what (7.17) says. Moreover, we can see that the question of whether \( U(x, \theta_{-\Delta}) \) is decreasing or not
depends on the 'race' between the mean of $f(\theta | x, \theta_{-\Delta})$ and the point $\psi(x)$ as $x$ increases. If the mean is increasing faster than the point $\psi(x)$, then we indeed have decreasing $U$. In general, this cannot be guaranteed. However, we saw above that when $\epsilon$ is sufficiently small, it can be.

4.3 Example

We illustrate the effect of shifts in $\epsilon$ by means of a numerical example. Let $\Delta = 1$, $\theta = 0$, $\bar{\theta} = 1$, and $\theta_{-\Delta} = 0.5$. We plot the function $\psi(x)$ and the posterior mean $(x + \epsilon \theta_{-\Delta})/(1 + \epsilon)$ for four values of $\epsilon$. In figure 7.4a–d, the horizontal axis measures $x$, while the vertical axis measures $\theta$. The posterior mean is the straight line, while $\psi(x)$ is the curve.

As is clear from these plots, even for moderately large values of $\epsilon$, the slope of the posterior mean is steeper than the slope of $\psi$, so that $U$ is monotonic. Only when $\epsilon$ is very large (certainly larger than 5) do we have the possibility of $\psi$ being steeper than the posterior mean. These simulations
suggestion that the possible multiple equilibria resulting from a non-monotonic $U$ function may not be important in practice.

From lemmas 1 and 3, theorem 1 follows from the following argument.

For sufficiently small $\varepsilon$, we have a unique point $x'(\theta_{-\Delta})$ at which $U(x, \theta_{-\Delta})$ cuts the horizontal axis. Hence, in every equilibrium, every speculator attacks given $(x, \theta_{-\Delta})$ if $x \leq x'(\theta_{-\Delta})$. Then, consider the value of $\theta$ given by
$\psi(x(\theta_{-\Delta})), \text{ where } \psi \text{ is the function defined in (7.16). The hurdle process }$
\{h(t)\} \text{ is defined to be the stochastic process such that }$
\[ h(t) = \psi(x(t - \Delta)). \]
(7.28)

To prove theorem 2, note from (7.17) that $U(x, \theta_{-\Delta})$ is decreasing in $\theta_{-\Delta}$ since an increase in $\theta_{-\Delta}$ induces a rightward shift in the posterior density $f$. 
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(· | x, θ−δ). This, in turn implies a lower value of x∗(θ−δ), and hence a lower value of ψ(x∗(θ−δ)). This completes the proof. ■

5 The limiting case

A case of particular interest to us is the limiting case when the noise ε tends to zero. This serves as a benchmark in several respects. Since we envisage ε as being very small, the limit gives us an indication of the likely shape of the various quantities we have been working with, in particular the function U. Indeed, the U function has a particularly simple characterisation in terms of the pain threshold function a(.)

Theorem 3 \( \lim_{ε→0} \partial U/\partial x = -a' \).

Since U lies between 1 − c and −c, theorem 3 determines the U function uniquely as the 'upside-down' version of the function a(·), where its level is fixed to lie between 1 − c and c. To prove this result, let us use the shorthand of

\[ \phi = \phi \left(x, \psi(x), \sqrt{εΔ}\right), \]

and \( \phi' = \phi \left(\psi(x), \frac{x + εθ_{δ}}{1 + ε}, \frac{εΔ}{1 + ε}\right) \)

then we have

\[ \frac{\partial U}{\partial x} = \phi' \left[ \frac{\phi}{a' + \phi} - \frac{1}{1 + ε} \right] \]

\[ = \left[ \frac{\phi}{(1 + ε)(a' + \phi)} \right] (εφ - a'). \]

In the interval \((θ, θ̅)\), these densities become degenerate as ε becomes small, so that the expression in square brackets tends to 1, while εφ → 0. Hence,

\[ \frac{\partial U}{\partial x} \to -a'. \quad (7.29) \]

This is a very appealing result, in that it gives a simple characterisation of the U function in terms of the fundamentals of the problem. The shape of the U function in the limit is the mirror image of the ‘pain threshold’ function a. This puts the focus squarely on the factors which determine the a function.

For instance, if the interval \((θ, θ̅)\) is wide, then the slope of a is shallow, and a small increase in cost has large impact on the cutoff θ'. In terms of the economic interpretation, a wide interval \((θ, θ̅)\) translates into the
statement that speculators’ actions are more influential/decisive in dictating the exchange rate. A variety of factors will influence such decisiveness. The size of a country relative to the pool of ‘hot’ money will certainly be a factor, as well as the composition of financial flows and the maturity structure of debt. A shallow \( a(.) \) function can also be seen reflecting the strength of the mutually reinforcing nature of the actions undermining the peg.

6 Concluding remarks

While the theory advanced in this chapter is too rudimentary to serve as a tool for assessing practical policy alternatives, it does set out the considerations which could guide our thinking. We regard the contribution here very much as a conceptual one. We believe that our approach provides a handle on the evolution of beliefs which trigger the change of sentiment, which in turn precipitates the attack. In this sense, we propose our theory as one of the onset of currency attacks. Developments of this framework may shed further light on the problem.

NOTES

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1. An excellent bibliography on the Asian financial crisis is maintained by Nouriel Roubini on http://www.stern.nyu.edu/~nroubini/asia/AsiaHomepage.html


3. The Roubini bibliography cited earlier contains a comprehensive list. A paper with suggestive results is Kumar, Moorthy and Perraudin (1998).

4. This fixed-point characterisation of common knowledge was first given formal treatment by Aumann (1976), and emphasised in Barwise (1988), Shin (1993) and others. Monderer and Samet (1989) discuss the analogous fixed-point characterisation of common \( p \)-belief.

REFERENCES


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Chapter 7 takes further the ideas of Morris and Shin (1998) in that it allows for more natural Brownian-motion representation of fundamentals. In addition the chapter assumes that the information of market participants about the true value of the fundamental is distributed according to a normal distribution about the true value, as opposed to a uniform distribution in the earlier paper. This allows the authors to tell a dynamic story about when a currency attack might occur, and also to examine what happens when the nature of the differential private information is varied. The results remain startling: a tiny departure from common knowledge entirely eliminates the multiplicity of equilibria. This work represents a major challenge to the traditional multiple-equilibrium models of currency attacks, most notably associated with Obstfeld (1986, for example). Of course the insights here are also applicable to other market situations with similar multiple-equilibrium properties, such as bank-run models.

The arguments of the chapter, however, are technical and it is difficult to get a clear idea of why a small departure from common knowledge about the fundamental can lead to such a radical change in the set of equilibria. So I will start by attempting an overview of the argument, before discussing the extent to which I think these insights are important.

Rather than arguing that it is possible to rule out all but one arbitrary equilibria, I shall present their argument in terms of a single but natural class of equilibria. Recall some of the essential ingredients of the model: the lagged value of the fundmental $\theta$ is known to be $\theta_{-\Delta}$, and the typical speculator receives an idiosyncratic signal about the current value of $\theta$ equal to $x$. She then has to decide whether to attack the currency (sell short) or not. The attack succeeds only if the proportion of speculators attacking exceeds $a(\theta)$, where $a(\theta)$ is an increasing function. In what follows $\theta_{-\Delta}$ should be regarded as being fixed. Consider then equilibria of the following form, which we shall call ‘cutoff speculation rules’: ‘attack the currency if the signal received ($x$) is no greater than a critical $\hat{x}$.’ While there are many other types of conceivable equilibria, this is the most natural class and we shall see that the argument extends easily to all other putative equilibria.

First, assume that there is no differential information so that it is common knowledge that each speculator knows $\theta$ (i.e. $x = \theta$). Then the multiplicity of equilibria within this class is demonstrated by the fact that any $\hat{x}$ between $\theta$ and $\bar{\theta}$ will do for an equilibrium cutoff speculation rule. That is, under complete information, all speculators can coordinate their decisions perfectly and provided everyone else is attacking, it is optimal for
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each individual to attack since for \( \theta \in (\tilde{\theta}, \bar{\theta}) \), the monetary authority will not defend the peg if everyone attacks, while if no one else is attacking it is optimal not to attack since the authority will defend. Clearly, then, this is an equilibrium if \( \hat{x} \in (\tilde{\theta}, \bar{\theta}) \). It is important, for the line of thinking used by Morris and Shin, to think of the multiplicity of equilibria in this model not in terms of there being multiple equilibria once \( \theta \) is known (which, of course, there are) but, as above, in terms of there being a continuum of \textit{ex ante} equilibrium speculation rules, parametrised by \( \hat{x} \).

Now introduce the noise, so that \( x \) is a noisy signal of \( \theta \). We ask the same question: which cutoff speculation rules are consistent with equilibrium? Naturally the problem is now much more complicated for the individual speculator: knowing only her own signal \( x \), she cannot know whether other speculators' signals are above or below \( \hat{x} \), nor can she know the true value of \( \theta \) and hence the critical size of attack needed to bring down the peg. Fortunately to check whether a particular \( \hat{x} \) works, it is necessary only to analyse the problem faced by the 'critical speculator' who receives the signal \( \hat{x} \). This speculator must be indifferent about attacking or not. This is clear: if she strictly preferred to attack, then by continuity a speculator with \( x \) slightly above \( \hat{x} \) would also want to attack, which is inconsistent with the putative equilibrium which says that the latter speculator should not attack. A similar argument establishes that the critical speculator cannot strictly prefer not to attack. So the critical speculator must be indifferent, and a speculator with \( x < \hat{x} \) will have a higher conditional utility from attacking as she believes that it is more likely that \( \theta \) is lower, which both implies that the authority is less willing to defend, and implies that more speculators will attack (since the distribution of signals is shifted to the left, while \( \hat{x} \) remains constant). So all those who are supposed to attack according to the speculation rule will want to. Likewise, those receiving signals above \( \hat{x} \) will strictly not want to attack. So the indifference of the critical speculator is necessary and \textit{sufficient} for a particular \( \hat{x} \) to be an equilibrium.\(^2\)

It should be intuitively clear that the continuum of equilibrium cutoff rules in the complete information environment will not survive the introduction of noise: an equilibrium now depends on finding an \( \hat{x} \) for which the indifference condition exactly holds. Since it involves a calculation of a non-trivial conditional expectation, there is absolutely no reason why it should not hold at any particular value of \( x \). (By contrast, with complete information, perfect coordination was possible and there was no such condition that needed to be checked.) This does not establish uniqueness, however. To do that, it is necessary to show that there is a single \( \hat{x} \) for which the indifference condition holds. It seems quite intuitive that there should be only a single indifference point. If there were two such values – say, \( \hat{x}_1 \) and \( \hat{x}_2 \), with \( \hat{x}_1 < \hat{x}_2 \) – then the critical speculator under the rule associated with
\( \hat{x}_1 \) would be receiving a lower signal (\( \hat{x}_1 \)) than the critical speculator under the \( \hat{x}_2 \) rule, and would consequently expect \( \theta \) to be lower (i.e. the mean of \( f(\theta; \hat{x}, \theta_{-\Delta}) \) would be lower). This conditional expectation will not be lower by as much as \( \hat{x}_2 - \hat{x}_1 \), however, as it is a weighted average of \( x \) and \( \theta_{-\Delta} \), and will not fully respond to the change in the signal. Now for each point in the conditional density \( f(\theta; \hat{x}, \theta_{-\Delta}) \), for \( \hat{x}_1 \), this corresponds to a lower value of \( \theta \) which implies that the monetary authority is less resistant to attack, but at the same time because the density has shifted to the left by less than \( \hat{x}_2 - \hat{x}_1 \), that part of the signal density falling below \( \hat{x}_1 \) will be smaller – i.e. the size of the attack will be lower. As long as the latter effect is smaller than the former, then there will be more points in the density corresponding to successful attacks. In other words, the probability of a successful attack would increase as \( \hat{x} \) falls. Since the utility of attacking is just this probability less \( c \), this would imply that the critical speculator cannot be indifferent at both \( \hat{x}_1 \) and \( \hat{x}_2 \). Morris and Shin show that as \( \epsilon \) becomes small, the latter effect becomes insignificant and there will be a unique equilibrium.

The reason for this seems to be as follows: \( \Delta \) (the unit of time) is held constant, so the variance of the signal, \( \epsilon\Delta \), decreases, and the speculator will put less weight on the lagged value of the fundamental \( \theta_{-\Delta} \), and more on the signal \( x \) itself. Consider, again, for a fixed cutoff rule associated with \( \hat{x} \), the marginal speculator who receives the signal \( \hat{x} \). While the conditional mean of \( \theta, E[\theta; \hat{x}, \theta_{-\Delta}] \), will become very close to \( \hat{x} \), as \( \epsilon \to 0 \), the variance of both the conditional distribution of \( \theta \) and of the signal distribution for each possible value of \( \theta \) declines, and it is not obvious how this affects the probability that there will be sufficient signals falling below \( \hat{x} \) to bring down the peg. Suppose that \( \hat{x} < \theta_{-\Delta} \). Note that the speculator receiving this signal \( \hat{x} \) must believe that it is more likely that \( \theta \) lies above than below \( \hat{x} \). From the expression below (7.4) in the chapter (p. 239), for \( \epsilon \) small

\[
E[\theta; \hat{x}, \theta_{-\Delta}] - \hat{x} = \epsilon(\theta_{-\Delta} - \hat{x})
\]

and consider the distance \( d \) such that a fraction \( k < 0.5 \) of the conditional density lies below \( E[\theta; \hat{x}, \theta_{-\Delta}] - d \); this is proportional to the standard deviation of this distribution

\[
\sqrt{\epsilon\Delta/(1 + \epsilon)}
\]

i.e. \( d = K(k)\sqrt{\epsilon\Delta} \) for \( \epsilon \) small. As

\[
\epsilon \to 0, \epsilon(\theta_{-\Delta} - \hat{x}) < K(k)\sqrt{\epsilon\Delta},
\]

so that for any \( k < 0.5 \), there is a small enough \( \epsilon \) such that the critical speculator believes that the probability that \( \theta < \hat{x} \) is greater than \( k \). Consequently, as the precision of the signal becomes very large, the speculator receiving the signal \( \hat{x} \) (or indeed any signal) will reckon that it is
equally likely that the true value of the fundamental, \( \theta \) lies below or above \( \hat{x} \). The crucial point, though, is that the lagged value of \( \theta \) does not matter in the calculations of the critical speculator – that is, her beliefs about the signal distribution relative to \( \hat{x} \) do not depend on where \( \hat{x} \) happens to be; since, however, at lower values of \( \hat{x} \), \( \theta \) and hence the proportion of speculators needed to successfully attack is likely to be smaller, the anticipated payoff of the critical speculator must be higher than at a putative equilibrium with a higher \( \hat{x} \). The indifference condition cannot be satisfied at more than one \( \hat{x} \).

In some ways, the importance of the result is not so much the uniqueness result per se, although of course it is very striking, but the fact that even a small departure from common knowledge destroys the coordination amongst speculators and requires a completely different analysis. And their story for the lack of common knowledge in terms of fragmentation of information is surely very compelling.

It is important, though, to realise the way of looking at the problem that Morris and Shin adopt. One might think about a possible multiple-equilibrium story as follows: the economy is currently sitting in some good equilibrium when a piece of unexpected bad news turns up which might precipitate a shift to a bad equilibrium. In the Morris–Shin version of this, speculators have already thought about all the possible pieces of bad news that could possibly turn up and have a clear idea of what the critical piece of bad news would be that would induce them to assume that the bad equilibrium is appropriate. This does require a great deal of rationality and contingent planning on the part of speculators.

The model, while rudimentary, also seems to have quite strong empirical implications. In the first place, as they point out, it at least has empirical implications, whereas this is debatable in the case of multiple-equilibrium models in the absence of any compelling account of equilibrium selection. Secondly, if the continuous-time approximation is taken seriously (\( \Delta \) very small) it implies that provided that \( \theta \) is not close to the ‘hurdle’ process, the probability of the peg collapsing in the next period is very close to zero. As \( \theta \) gets very close to the hurdle, this probability will suddenly become very large. It may be possible to see from estimates of risk premia whether this sharp spike in the probability away from zero is reasonable. Devaluation probabilities which were small but positive and reasonably constant over time would seem to be prima facie evidence against at least the simple Shin–Morris model. Indeed, Sutherland (1997) shows how the term structure of interest rates can be used to differentiate between multiple-equilibrium and fundamentals-based crisis models. The latter will lead to very similar implications as the Morris–Shin model.

The model is very stylised. As a dynamic model it may be somewhat
inconsistent. For example, each period there will be some speculators who unsuccessfully speculate, and who thus lose c, but the model assumes their continued participation. It is difficult to think of a more fully specified model in which each speculator can sell short at most one unit of currency per period (credit-constrained?), and in addition this is independent of how many times in the past the speculator has been unsuccessful. Likewise, the monetary authority is assumed to defend the peg whenever the size of attack is insufficiently large. This means that each period it will have to defend against some attack; yet a(θ) is assumed to be independent of the past history of attacks. Again, this assumption appears to be rather strong; it would be interesting to know to what extent such features of the model are critical to the conclusions.

Nevertheless, the chapter represents a fundamental challenge to the way we think about multiple-equilibrium models, not just of currency attacks, but in a wide range of situations.

NOTES

1. See n. 3 below. Although Morris and Shin do not present the argument explicitly in this way, their argument essentially says that one can restrict attention without loss of generality to this class of strategies.

2. The indifference condition is just the condition \( U(\hat{x}, \theta_{-}\hat{x}) = 0 \) of the chapter, as \( U(\hat{x}, \theta_{-}\hat{x}) \) is defined to be the utility from attacking for a critical speculator in the cutoff speculation rule associated with \( \hat{x} \).

3. Again we are restricting attention to cutoff rule equilibria. However this is without loss of generality: suppose it is true that there is a unique cutoff rule equilibrium at \( \hat{x} \). If there were some other type of equilibrium, consider the lowest (infimum) \( x' \) (say, \( x' \)) such that speculators do not attack with probability one; if this was below \( \hat{x} \) then a speculator receiving the signal \( x' \) would be in a better position than the critical speculator in a cutoff rule at \( x' \) because of the fact that there are now additional speculators (by virtue of the fact that this is not a cutoff rule equilibrium), with higher signals than \( x' \) who attack, and this can only help the attack ("strategic complementarity", lemma 2, p. 246). Since the critical speculator in a cutoff rule at \( x' \) will be shown to have a positive utility from attacking, this is true a fortiori for the \( x' \) speculator in this other equilibrium, contradicting the fact that this speculator does not attack (or at least must be indifferent). So \( x' = \hat{x} \). A symmetric argument establishes the highest \( x \) such that an attack takes place with positive probability is \( \hat{x} \). Hence there can be no other equilibrium (this is just lemma 3, p. 248).

4. To be rather more precise, the standard deviation of the signal distribution, given \( \theta \), is \( \sqrt{\epsilon \Delta} \), and so the speculator knows for \( \epsilon \) small that the variance of the signal about \( \theta \) is the same as the variance of \( \theta \) about \( \hat{x} \). Hence a successful attack, which requires roughly a fraction \( a(\hat{x}) \) of speculators to attack (i.e. a fraction \( a(\hat{x}) \) of signals to be below \( \hat{x} \)), will happen with probability \( 1 - a(\hat{x}) \) – this simply follows from the symmetry of the normal distribution. Since the indifference condition
requires that this probability equal \( c \) (the cost of attacking), and \( a' > 0 \), the condition can be satisfied only at a single point. In fact this is what lemma 2 states: the utility for the critical speculator from attacking as \( \bar{x} \) varied as slope \(-a' (\bar{x})\).

5. Thus introducing further sources of non-uniqueness into the model may alter the conclusions substantially. Boonprakaikawe (1998) shows that in a formally similar model of bank runs with a monopoly bank, an analogous argument for uniqueness holds in the absence of common knowledge of the fundamental. With two banks competing for deposits, however, there may still be multiple equilibria. The cause is an externality operating between the banks: each bank’s failure probability is increasing in the number of depositors attracted by the other bank. The lack of common knowledge of the fundamental does not remove the non-uniqueness owing to this externality.

6. A switch in equilibrium to a coordinated attack is modelled by a Poisson process.

REFERENCES