1 Introduction

Supergiant X-ray binary (SgXB) consists of an accreting compact object, often a neutron star (NS), and a supergiant O/B star. They are among the brightest X-ray sources in our Galaxy. For a small fraction of SgXBs, the supergiant companion fills its Roche lobe and the overflowing stellar material forms an accretion disk around the compact object (Soberman et al., 1997). For other SgXBs, the compact object accretes from the fast stellar wind of the companion. The supergiant companion can lose up to several $10^{-6} M_\odot/yr$ in stellar wind, and a small fraction of this ends up being accreted. The two scenarios above can be called disk-fed and wind-fed, respectively. In this paper we focus on the more common wind-fed SgXBs, and assume that the compact object is a NS.

Understanding the morphology of the accretion flow (especially, whether a disk can form around the NS) is crucial. It affects the mass and angular momentum accretion rate, which then determines the X-ray luminosity, variability, and spin evolution. It also determines what kind of model for small-scale (near and inside the magnetosphere) accretion dynamics is appropriate. Currently, most models assume accretion from a thin disk (Shakura and Sunyaev, 1973) or a quasi-spherical (and laminar) inflow (Shakura et al., 2013); but as we will show later, these assumptions are often invalid.

For one particular system, OAO 1657-415, observations favor the existence of a disk. Its spin evolution suggests sporadic accretion from a disk (Jenke et al., 2012), and cyclotron line observation suggests a magnetic field strength inconsistent with a disk-less wind-fed scenario (Taani et al., 2018). However, for most observed systems there is no conclusive observational clue regarding the existence of a permanent disk (or disk-like structure) around the NS (Shakura et al., 2012; some evidences of transient disk: Romano et al. 2015; Hu et al. 2017). Thus, theoretical modeling of the accretion flow is necessary in order to understand the accretion dynamics and better interpret observations.

The wind accretion process is often studied using the Bondi-Hoyle-Lyttleton (BHL) model (see a review in Edgar 2004). The standard BHL model considers a point mass accreting from a supersonic flow that is homogeneous with constant velocity at infinity. Gravitationally deflected, the flow develops a bow shock in front of the accretor and an overdense tail. This simple model is a relatively good
approximation at low NS mass (so that radiative acceleration of the wind near the NS is negligible) and fast upstream wind speed ($\gg$ orbital velocity). More generally, the upstream flow may be asymmetric, and such asymmetry can be modeled by imposing a transverse gradient (e.g., density or velocity gradient) on the upstream flow. This breaks the axisymmetry of standard BHL accretion, and gives the accretion flow a finite mean specific angular momentum. In wind-fed SgXBs, such transverse gradient is mainly due to misalignment between the wind velocity (in the frame rotating with the binary) and the direction of the companion, and is usually small (see §2.3).

Analytic and numerical studies of BHL accretion (with and without upstream gradient) have a long history (see review of earlier studies in Edgar 2004 and Foglizzo et al. 2005; some more recent studies are Blondin and Pope 2009; Blondin and Raymer 2012; Blondin 2013; MacLeod and Ramirez-Ruiz 2015). Simulations of 3D BHL accretion overall find that the accretion flow is more prone to instability at higher Mach number, smaller accretor size, and larger upstream gradient. Some important questions, however, remain unanswered:

- The mechanism of instability remains uncertain. Several possible instability mechanisms have been proposed, but none is confirmed to be the main reason of instability (see discussion in Foglizzo et al. 2005).
- The boundary (in parameter space) between stable and unstable accretion flow is unclear. No individual study has covered both sides of this boundary with sufficient number of simulations to produce a tight constraint. Knowing this boundary will also help determining the validity of proposed instability mechanisms.
- The observed systems tend to have accretor size (for NS, this would approximately be the size of the magnetosphere) smaller than what is achievable in simulations, and it is important to know whether the system's behavior converges as the accretor size decreases. This has not been answered by previous simulations, which usually include only one or two accretor sizes.
- The criterion of disk-formation is also uncertain. In analytic studies (e.g., Ho 1988), it is often assumed that disk forms when the mean specific angular momentum supplied by the upstream gradient approximately exceeds the Keplerian value at the magnetosphere, but the validity of this argument is dubious when the flow near the accretor is turbulent due to instability.

In this paper, we attempt to address the above questions by numerically constraining the boundary of instability, testing convergence using multiple accretor sizes, and discussing disk formation based on understanding of flow morphology and angular momentum transport. We focus on the regime with small accretor size and small upstream gradient, which is relevant to most SgXBs (see §2.3) but has not been systematically explored with simulations before.

Although BHL accretion with upstream gradient captures some of the main physics of wind-fed SgXB accretion, other effects are likely non-negligible. These include orbital effects, line-driven acceleration of the upstream wind (and the inhibition of which by NS radiation feedback), and radiative cooling near the NS. Some previous studies address these effects: Blondin et al. (1990) and Manousakis and Walter (2015) perform (planar) 2D simulations including orbital effects and realistic line-driven acceleration (with NS feedback); the latter successfully reproduces the off-states of Vela X-1; El Mellah et al. (2018a) perform 3D simulations including orbital effects and cooling, using an upstream boundary condition derived from a realistic wind model, and demonstrate the importance of orbital effect and cooling for disk formation. In this paper, we also perform 3D simulations including orbital effects and a parametric model of wind acceleration to compare with BHL results.

Our paper is organized as follows: In Section 2, we parameterize the stellar wind and estimate parameters relevant to accretion (especially the strength of upstream gradient) for a number of observed SgXBs. Section 3 introduces our numerical method. Section 4 covers 2D axisymmetric simulations of BHL accretion; we discuss flow stability and convergence with respect to accretor size and resolution. Section 5 presents 3D simulations of BHL accretion with upstream gradient; we cover a wide range of accretor size and upstreamgradient, and discuss flow morphology and mechanism of instability. In Section 6, we develop a model including orbital effects and parameterized wind acceleration, and apply it to observed systems. In Section 7, we discuss angular momentum transport and disk formation, and summarize regimes of different behaviors in parameter space. We present our conclusions in Section 8.

2In 2D (planar) BHL accretion, spontaneous disk formation has been observed when there is no upstream gradient (Blondin and Pope, 2009; Blondin, 2013), but similar behavior is not observed in 3D (MacLeod and Ramirez-Ruiz, 2015).
2 System parameters

In this section, we start by introducing the normalization and stellar wind model we adopt, then apply the wind model to observed systems to obtain a series of parameters relevant for the accretion process.

2.1 Normalization

Consider a NS with mass $M_{NS}$ accreting from a wind with density $\rho_\infty$ and velocity $v_\infty$ at infinity. A natural length unit of the problem is the accretion radius $R_a$ defined as

$$R_a = \frac{2GM_{NS}}{v_\infty^2}. \tag{1}$$

$R_a$ is also the length scale within which the flow is significantly affected by the gravity of the accretor. In this paper, we use the normalization $GM_{NS} = R_a = \rho_\infty = 1$. Under this normalization, $v_\infty = \sqrt{2}$ and the unit time is the accretion time $t_a$ defined as

$$t_a = \sqrt{\frac{R_a^3}{GM_{NS}}} = \frac{\sqrt{2}R_a}{v_\infty}. \tag{2}$$

$t_a$ is roughly the time it takes for the flow to cross $R_a$.

2.2 Stellar wind model

In reality, the NS in a SgXB accretes from the wind of the companion star. Here we propose a simple parametrization of the single-star (i.e. with NS gravity ignored) wind profile of the companion. This single-star wind model can help quantify how wind accretion in real systems differs from the ideal BHL accretion.

Consider a non-rotating spherical-polar coordinate $(R, \Theta, \Phi)$ centered at the companion. [We reserve $(r, \theta, \phi)$ for coordinate centered at the NS.] For simplicity, we assume that the binary orbit is circular with semi-major axis $a_b$ and the spin of the companion is aligned with the orbital angular momentum. When the companion is not spinning, the radiative driving force produces a steady state radial velocity profile approximately given by

$$v_R = v_t \left(1 - \frac{R_c}{R}\right)^\beta, \tag{3}$$

with $v_t$ being the terminal velocity of the wind, $R_c$ the radius of the companion, and $\beta \approx 0.8$ (Friend and Abbott, 1986). Conservation of mass then implies that the density of the wind is

$$\rho = \frac{M_{\text{wind}}}{4\pi R^2 v_R}. \tag{4}$$

Here $M_{\text{wind}}$ is the wind mass loss rate. We also assume that the wind is isothermal in steady-state, with temperature equal to the effective temperature at the stellar surface $T_{\text{eff}}$. Therefore, the sound speed is constant for the single-star wind.

We add in the effect of stellar spin by assuming $\Theta$-independent radial velocity, zero azimuthal velocity, velocity continuity at the stellar surface and conservation of angular momentum. Under these assumptions, $v_R$ is still given by (3) and

$$v_\Theta = 0, \quad v_\Phi = \frac{\Omega_c R_c^2 \sin \Theta}{R}. \tag{5}$$

Here $\Omega_c$ is the spin frequency of the companion star.

When analyzing the accretion flow, it is convenient to work in a frame rotating with the binary’s orbital frequency $\Omega_b$ so that the binary remains fixed. Since the companion is significantly more massive than the NS, $v_R, v_\Theta$ approximately remain the same in this frame, and $v_\Phi$ becomes

$$v_\Phi = \frac{\Omega_c R_c^2 \sin \Theta}{R} - \Omega_b R \sin \Theta. \tag{6}$$
2.3 Parameters of observed systems

Relevant orbital and wind parameters of several observed systems are summarized in Table 1. From these parameters, we can derive a set of parameters more directly related to accretion, which are given in Table 2. Below, we discuss the significance of some of these parameters.

The accretion radius \( R_a \) is \( 10^{-11} \) cm for all systems, with a spread of approximately one order of magnitude. This is much greater than the size of the NS (\( \sim 10^6 \) cm), and resolving the NS in any 3D simulation covering a few \( t_a \) is highly unfeasible due to the extremely short timestep. Meanwhile, the magnetosphere can be more than a factor of 100 larger, with (Davidson and Ostriker, 1973)

\[
R_{\text{mag}} \approx 2.6 \times 10^8 \text{cm} \left( \frac{B_0}{10^{12} \text{G}} \right)^{4/7} \left( \frac{R_{\text{NS}}}{10 \text{km}} \right)^{10/7} \left( \frac{M_{\text{NS}}}{M_{\odot}} \right)^{1/7} \left( \frac{L_x}{10^{37} \text{erg/s}} \right)^{-2/7}.
\]  

(7)

Here \( B_0 \) is the surface magnetic field, and \( L_x \) the luminosity of the NS. Depending on the strength of magnet field and the accretion rate (which determines \( L_x \)), \( R_{\text{mag}} \) can sometimes be \( \gtrsim 10^9 \) cm. To some extend, \( R_{\text{mag}} \) can be considered as an effective size of the accretor, since within \( R_{\text{mag}} \) the magnetic field can help remove excessive angular momentum from the flow, allowing efficient accretion. Resolving \( R_{\text{mag}} \approx 10^{-2} - 10^{-3} R_a \) is feasible for many systems.

The azimuthal velocity \( v_\phi \), which is due to orbital motion and companion rotation, is often comparable to the radial velocity \( v_R \). Therefore, the direction of the upstream wind (as well as the shock and the overdense region behind which can affect spectral features) is significantly misaligned with respect to the direction of the companion, and ignoring the orbital motion is usually not a good approximation.

The Mach number \( M \) of the systems all lie in the regime of high Mach number (\( M \gtrsim 10 \)). As we will show later, in this regime, the dynamics of the accretion flow is not sensitive to the Mach number, since the internal energy in the upstream flow is already negligible.

The transverse gradient parameters, \( \epsilon_\rho \) and \( \epsilon_v \), are defined by

\[
\epsilon_\rho = R_a \frac{\partial \ln \rho}{\partial y}, \quad \epsilon_v = R_a \frac{\partial \ln v_x}{\partial y}.
\]  

(8)

Here \((x, y, z)\) is the cartesian coordinate with the \( xy \) plane being the orbital plane, \(+z\) aligned with the orbital angular momentum, and \(-x\) aligned with the (rotating frame) wind velocity \( \mathbf{v} \) at NS. \( \epsilon_\rho, \epsilon_v \) approximately correspond to the fractional change of \( \rho, v \) per \( R_a \), and directly characterize the strength of transverse gradient in the upstream flow. For all systems, \( \epsilon_\rho \) and \( \epsilon_v \) have opposite sign (i.e. the side with higher density has lower velocity). The density gradient is the main gradient, with \( \epsilon_\rho \) larger than \( |\epsilon_v| \) by a factor of a few. In general, \( \rho, v \) should also have a gradient along the direction of the flow, but such gradient is less important since it does not break the symmetry (e.g. the flow can still be axisymmetric with a gradient along the flow).

The strength of transverse upstream gradient shows a large scatter among systems, with \( \epsilon_\rho \) ranging form \( < 0.01 \) (e.g. 4U 1907+097) to \( \sim 0.5 \) (OAO 1657-415). As a result, these systems may exhibit qualitatively different behaviors (e.g. with or without the formation of a disk-like structure). Most previous studies (e.g. MacLeod and Ramirez-Ruiz 2015; El Mellah et al. 2018a) cover the regime of \( \epsilon_\rho \gtrsim 0.1 \); in this paper, we will mainly explore the regime of smaller upstream gradient, \( \epsilon_\rho \lesssim 0.1 \), in order to provide better coverage for the parameter space relevant for wind accretion in SgXB systems.

Finally, the ratio \( R_a/R_H \) (with \( R_H \) being the size of the Hill sphere) and the parameter \( \Omega_b t_a \) characterize the importance of accelerations related to orbital effects. \( R_a/R_H \) characterizes the strength of the companion’s gravity. Most systems have \( R_a < R_H \), i.e. the companion’s gravity is unimportant.

The only exception is OAO 1657-415, which has \( R_a \approx R_H \). \( \Omega_b t_a \) characterizes the strength of Coriolis force in the rotating frame; it is approximately the ratio between Coriolis force and NS gravity at \( \sim 1 R_a \). Meanwhile, the ratio between centrifugal force in the rotating frame and NS gravity at \( \sim 1 R_a \) is \( \sim (\Omega_b t_a)^2 \). All systems have \( \Omega_b t_a < 1 \), so centrifugal force is weaker than Coriolis force, and Coriolis force is weaker than NS gravity. Still, \( \Omega_b t_a \) is often comparable to \( \epsilon_v \), meaning that the Coriolis force may be as important as the transverse upstream gradients.

Among the many physical parameters that affect the strength of transverse upstream gradient (\( \epsilon_\rho, \epsilon_v \)) and the importance of orbital effects (\( R_a/R_H, \Omega_b t_a \)), the wind speed is the most important one.\(^3\) For high wind speed, \( R_a \) and \( t_a \) are small, making orbital effects unimportant. Larger \( v_R \) and smaller \( R_a \)

\(^3\)This is mainly because it shows more variation compared to other parameters. There are only three relevant dimensionless parameters (if we ignore the companion’s rotation), which are \( M_{\text{NS}}/M_c, a_b/R_c \) and \( v_t/v_{\text{orb}} \) (where \( v_{\text{orb}} \) is the NS orbital velocity). Among them, \( v_t/v_{\text{orb}} \) shows the largest variation across systems, since the terminal velocity \( v_t \) is sensitive to the temperature of the companion.
also give smaller transverse upstream gradient, since \( \epsilon_{\rho,v} \propto R_a v_\Phi/v \). Therefore, at high wind speed, accretion can be modeled using standard BHL accretion. On the other hand, at low wind speed (e.g. OAO 1657-415), upstream gradients and orbital effects are crucial and simple BHL model may not apply.
<table>
<thead>
<tr>
<th>System</th>
<th>$M_{\text{NS}}$ [$M_\odot$]</th>
<th>$M_c$ [$M_\odot$]</th>
<th>$R_c$ [$R_\odot$]</th>
<th>$P_b$ [d]</th>
<th>$P_c$ [d]</th>
<th>$P_{\text{NS}}$ [s]</th>
<th>$e$</th>
<th>$\dot{M}<em>{\text{wind}}$ [$M</em>\odot$/yr]</th>
<th>$v_t$ [km/s]</th>
<th>$T_{\text{eff}}$ [$10^4$K]</th>
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<tr>
<td>4U 1538-522</td>
<td>1.02</td>
<td>16</td>
<td>13</td>
<td>3.728</td>
<td>-</td>
<td>526.8</td>
<td>0.18</td>
<td>$8.3 \times 10^{-7}$</td>
<td>(1000)</td>
<td>(2.8)</td>
</tr>
<tr>
<td>4U 1700-377</td>
<td>1.96</td>
<td>46</td>
<td>22</td>
<td>3.412</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$&gt;2.1 \times 10^{-6}$</td>
<td>1700</td>
<td>4.2$^a$</td>
</tr>
<tr>
<td>4U 1907+097$^b$</td>
<td>-</td>
<td>27</td>
<td>26</td>
<td>8.375</td>
<td>-</td>
<td>440</td>
<td>0.28</td>
<td>$7 \times 10^{-6}$</td>
<td>1700</td>
<td>3.05</td>
</tr>
<tr>
<td>EXO 1722-363</td>
<td>1.91</td>
<td>18</td>
<td>26</td>
<td>9.740</td>
<td>-</td>
<td>413.9</td>
<td>&lt;0.19</td>
<td>$9.0 \times 10^{-7}$</td>
<td>(650)</td>
<td>(2.5)</td>
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<tr>
<td>GX 301-2$^c$</td>
<td>2.0</td>
<td>43</td>
<td>62</td>
<td>41.37</td>
<td>63</td>
<td>690</td>
<td>0.46</td>
<td>$1 \times 10^{-5}$</td>
<td>305</td>
<td>1.81</td>
</tr>
<tr>
<td>OAO 1657-415</td>
<td>1.74</td>
<td>17.5</td>
<td>25</td>
<td>10.45</td>
<td>12$^d$</td>
<td>37.3</td>
<td>0.10</td>
<td>$(1.1 - 5.6) \times 10^{-7}$</td>
<td>250</td>
<td>2.0$^d$</td>
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<td>SAX J1802.7-2017</td>
<td>1.57</td>
<td>22</td>
<td>18</td>
<td>4.570</td>
<td>-</td>
<td>139.6</td>
<td>-</td>
<td>$6.3 \times 10^{-7}$</td>
<td>(680)</td>
<td>(2.0)</td>
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<tr>
<td>Vela X-1 (slow)</td>
<td>2.12</td>
<td>26</td>
<td>29</td>
<td>8.964</td>
<td>6.0$^c$</td>
<td>283.2</td>
<td>0.09</td>
<td>$(1.0 - 5.3) \times 10^{-6}$</td>
<td>600</td>
<td>2.5</td>
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<tr>
<td>Vela X-1 (fast)$^f$</td>
<td>4.0$^f$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4 $\times 10^{-6}$</td>
<td>-</td>
<td>-</td>
<td>1700</td>
<td>2.5</td>
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<tr>
<td>XTE J1855-026</td>
<td>1.41</td>
<td>21</td>
<td>22</td>
<td>6.072</td>
<td>-</td>
<td>360.7</td>
<td>0.04</td>
<td>$(0.2 - 1.1) \times 10^{-5}$</td>
<td>(620)</td>
<td>(3.0)</td>
</tr>
</tbody>
</table>

Table 1: Physical parameters of several wind-fed SgXBs with relatively well-determined stellar properties. Subscript NS and c denote the NS and the companion respectively. $P_b$ is the binary orbital period, and $P_{\text{NS}}, P_c$ the spin period of the NS and the companion. $v_t$ is the terminal wind velocity. Bracketed data are estimated. There are two rows for Vela X-1 since its companion’s temperature lies close to the bifurcation between slow and fast winds, and earlier (fast) and more recent (slow) modeling produce very different $v_t$. References: [a] Heap and Corcoran 1992; [b] Cox et al. 2005; [c] Kaper et al. 2006; [d] Mason et al. 2012; [e] Quaintrell et al. 2003; [f] Nagase et al. 1986. All other data (including estimated $v_t$) come from Falanga et al. (2015) and references therein.
Table 2: Calculated accretion parameters for SgXBs in Table 1. When there is no observational data, we assume $M_{NS} = 1.4 M_\odot$ and the companion is non-rotating. (Assuming that the companion is rotating with $P_c = P_b$ gives qualitative similar parameters.) The velocity $(v_R, v_\Phi)$ is evaluated in the rotating frame in which the binary is fixed. $D = a_b - R_c$ is the separation between the NS and the surface of the companion (assuming circular orbit). $\epsilon_\rho, \epsilon_v$ are the transverse gradient of density and velocity for single-star wind profile at the NS, and physically correspond to the fractional change of $\rho, v$ per $R_a$ (see text for detailed definition). $R_a/R_H$ characterizes the importance of the companion’s gravity (with $R_H$ being the Hill sphere radius), and $\Omega_b t_a$ characterizes the importance of Coriolis force. The last five parameters show how significantly the system differs from axisymmetric accretion.

<table>
<thead>
<tr>
<th>System</th>
<th>$R_a$ [cm]</th>
<th>$t_a$ [s]</th>
<th>$a_b/R_c$</th>
<th>$v_\Phi/v_R$</th>
<th>$R_a/D$</th>
<th>$\mathcal{M}$</th>
<th>$\epsilon_\rho$</th>
<th>$\epsilon_v$</th>
<th>$R_a/R_H$</th>
<th>$\Omega_b t_a$</th>
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</thead>
<tbody>
<tr>
<td>4U 1538-522</td>
<td>$6.2 \times 10^{10}$</td>
<td>$1.3 \times 10^3$</td>
<td>1.96</td>
<td>-0.61</td>
<td>0.071</td>
<td>24</td>
<td>0.051</td>
<td>-0.016</td>
<td>0.19</td>
<td>0.026</td>
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<tr>
<td>4U 1700-377</td>
<td>$6.4 \times 10^{10}$</td>
<td>$1.0 \times 10^3$</td>
<td>1.55</td>
<td>-0.68</td>
<td>0.076</td>
<td>26</td>
<td>0.052</td>
<td>-0.020</td>
<td>0.17</td>
<td>0.022</td>
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<tr>
<td>4U 1907+097</td>
<td>$3.5 \times 10^{10}$</td>
<td>$4.9 \times 10^2$</td>
<td>2.0</td>
<td>-0.32</td>
<td>0.019</td>
<td>35</td>
<td>0.0083</td>
<td>-0.0024</td>
<td>0.058</td>
<td>0.0042</td>
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<td>EXO 1722-363</td>
<td>$2.5 \times 10^{11}$</td>
<td>$8.0 \times 10^3$</td>
<td>1.93</td>
<td>-0.72</td>
<td>0.15</td>
<td>22</td>
<td>0.12</td>
<td>-0.038</td>
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<td>GX 301-2</td>
<td>$6.2 \times 10^{11}$</td>
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<td>-0.92</td>
<td>0.078</td>
<td>13</td>
<td>0.083</td>
<td>-0.026</td>
<td>0.31</td>
<td>0.053</td>
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<tr>
<td>OAO 1657-415</td>
<td>$7.3 \times 10^{11}$</td>
<td>$4.1 \times 10^4$</td>
<td>2.1</td>
<td>-1.36</td>
<td>0.39</td>
<td>11</td>
<td>0.44</td>
<td>-0.20</td>
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<tr>
<td>Vela X-1 (slow)</td>
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<td>1.86</td>
<td>-0.53</td>
<td>0.24</td>
<td>14</td>
<td>0.155</td>
<td>-0.068</td>
<td>0.58</td>
<td>0.132</td>
</tr>
<tr>
<td>Vela X-1 (fast)</td>
<td>$6.5 \times 10^{10}$</td>
<td>$9.9 \times 10^2$</td>
<td>1.86</td>
<td>-0.188</td>
<td>0.038</td>
<td>35</td>
<td>0.0094</td>
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<td>$5.8 \times 10^3$</td>
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<td>0.159</td>
<td>16</td>
<td>0.149</td>
<td>-0.050</td>
<td>0.37</td>
<td>0.069</td>
</tr>
</tbody>
</table>
3 Method

In this paper, we investigate a few related but different problems, including axisymmetric BHL accretion (§4), BHL accretion with small transverse upstream gradient (§5), and a more realistic scenario where wind acceleration, orbital motion of the NS and gravity of the companion are all considered (§6); we also use two different grid geometries (spherical-polar for axisymmetric accretion and cartesian for others).

3.1 Equations solved

We solve the Euler equations for an inviscid compressible ideal gas using Athena++, an extension of the grid-based Godunov code package Athena (Stone et al., 2008). The equations being solved are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{vv} + P \mathbf{I}) = -\rho \nabla \Phi,$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + P) \mathbf{v}] = -\rho \nabla \Phi \cdot \mathbf{v},$$

with the total energy density $E$ given by

$$E = \frac{P}{\gamma - 1} + \frac{1}{2}\rho \mathbf{v}^2.$$

Throughout this paper, we adopt an ideal gas equation of state with $\gamma = 5/3$ and ignore the self-gravity of the wind. For most simulations, we also ignore the gravity of the companion, and the gravitational potential is simply $\Phi = -GM_{NS}/r$. We also perform a few simulations including the gravity of the companion in a non-inertial frame rotating at the orbital frequency. In this case, $\Phi = -GM_{NS}/r - GM_c/R$ and $\nabla \Phi$ in (10) and (11) are replaced by $(\nabla \Phi \mathbf{a})$ with

$$\mathbf{a} = -2\Omega_b \times \mathbf{v} - \Omega_b \times (\Omega_b \times \mathbf{R}).$$

Since the mass ratio $M_{NS}/M_c$ is small, we assume that the distance to the companion $\mathbf{R}$ is approximately the displacement from the center of mass.

3.2 Boundary conditions

We divide the outer boundary of the domain into upstream and downstream regions. For boundary in the upstream region, we impose a pre-defined wind profile depending on the problem. For boundary in the downstream region, we use a free flow boundary condition, which copies the flow properties of adjacent cells into ghost cells outside the domain; this free flow boundary condition allows both inflow and outflow. When dividing the outer boundary into upstream and downstream regions, we require that there is only inflow in the upstream region, while there can be both inflow and outflow in the downstream region. To ensure that any inflow from the downstream region do not affect accretion, we require the domain to be sufficiently large so that (1) all accreted material enter the domain through the upstream region, (2) the subsonic region behind the shock is fully contained inside the domain (i.e. the outer sonic surface does not intersect the outer boundary), and (3) any streamline that originates from the downstream region cannot enter the subsonic region.

The inner boundary, located at $r_{in}$, physically corresponds to a surface across which all flow can be accreted (this requires an effective reduction of angular momentum for $r < r_{in}$ due to, for instance, magnetic interaction; in this case $r_{in}$ should be comparable to the magnetopause distance). We use two types of inner boundary conditions, absorbing and outflow. For absorbing boundary condition, we set the velocity to zero and density and pressure to very small values in cells with $r < r_{in}$. The outflow boundary condition is identical to the free flow boundary condition, except that $v_r$ in ghost cell is set to zero if the cell next to the boundary has $v_r > 0$ (i.e. inflow into the domain). The outflow boundary condition is not very well-defined for cartesian grid when resolution is relatively low, thus we use it only for spherical polar grid. We show in Section 4.2.3 that switching between the two inner boundary conditions does not affect the result significantly.

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4This is equivalent to requiring that material entering the domain from the downstream region remains supersonic and is never accreted, i.e. such material is never in causal contact with the accretor.
3.3 Initial condition

By default, we specify the initial condition using the pre-defined wind profile imposed on the upstream boundary. This sometimes produces discontinuities, which can be naturally removed after evolving the system for a few flow-crossing time. When there are multiple simulations with all parameters (e.g. $M$, upstream gradients) being identical except $r_{in}$ or resolution, we initialize simulations with smaller $r_{in}$ or higher resolution using the final state of a previous simulation with larger $r_{in}$ or lower resolution, provided that the previous simulation attains a (laminar or turbulent) steady-state. This is typically more efficient, since the flow of the new simulation can usually relax into a steady-state within a few $t_a$.

3.4 The H-correction

It is known that numerical artifacts can develop near a strong, grid-parallel shock (the “carbuncle” instability, see Quirk 1994), producing large fluctuation on the shock front which can significantly perturb the flow behind the shock. Our simulations are prone to this instability, since the shock immediately in front of the accretor is strong and approximately parallel to the grid, especially for a spherical-polar grid. We suppress this instability via a technique called the H-correction, which adds dissipations to the transverse fluxes when the shock is grid-aligned. This technique and its implementation has been discussed in Stone et al. (2008). We illustrate the necessity of the H-correction through an example in Section 4.2.4.

4 Axisymmetric BHL accretion

First we consider the problem of standard BHL accretion, where the upstream wind is axisymmetric with no transverse gradient. In this section, we assume that there is no perturbation breaking the axisymmetry, allowing us to investigate the problem using 2D axisymmetric simulations. This case has been studied in many previous works with 2D axisymmetric simulations, starting from Hunt (1971). A review of previous works on this topic is given in Foglizzo et al. (2005). All previous 2D axisymmetric simulations show a stable flow for $\gamma = 5/3$, with the only exception being Koide et al. (1991) which reports a dome-like, indented shock in front of the accretor due to the formation of a vortex when $M \geq 5$; this leads to oscillation of the shock front and a small fluctuation of the accretion rate. In addition to the Mach number dependence, the result of Koide et al. also suggests a dependence of stability on inner boundary size when compared to other studies, as they use the smallest inner boundary ($r_{in} = 0.015$ and 0.005) among all previous 2D axisymmetric simulations. The dependence of stability on inner boundary size is later confirmed by Blondin and Raymer (2012), whose 3D simulations at $M = 3$ show a stable flow for $r_{in} = 0.05$ and an unstable flow with a near-axisymmetric “breathing mode” for $r_{in} = 0.01$, leading to an accretion rate fluctuation with $\sim 10\%$ amplitude.

In this section, we report 2D simulations with higher resolution than any previous study at $M = 3$ and 10 and $r_{in} = 0.01$ - 0.04. We also discuss the convergence with respect to increasing resolution (which is also decreasing numerical dissipation) and the effect of changing the inner boundary condition. This allows a detailed comparison with previous works (§4.3).

4.1 Setup

We use a 2D ($r, \theta$) spherical-polar grid, with the accretor (NS) located at the origin. The wind comes from $\theta = 0$ direction, and has uniform density $\rho_\infty$ and velocity $v_\infty$ at $r \to \infty$. Given our normalization, $\rho_\infty = 1$ and $v_\infty = \sqrt{2}$. The sound speed at infinity $c_\infty$ is determined by specifying the Mach number $M \equiv v_\infty/c_\infty$.

4.1.1 Grid and resolution

The grids are evenly spaced in $\theta$ and $\log r$, with a lowest resolution of 96 cells in $\theta$ and 14 cells per factor of 2 in $r$. This choice gives $\delta_\theta/r \approx 0.05$ and $\delta_r \approx 0.03$ where $\delta_\theta, \delta_r$ are the size of the cell. We vary resolution from 1$\times$ up to 4$\times$ our lowest resolution to investigate numerical convergence of the result. As a reference, the 2D axisymmetric simulation by Pogorelov et al. (2000) is $\sim 1\times$ our standard resolution (they use a special radial grid, which compared to our choice has higher / lower resolution near / far

\footnote{Here we exclude simulations with a non-absorbing or not fully absorbing accretor, such as Fryxell et al. (1987) and Matsuda et al. (1989).}
from the accretor) and the 3D simulation by Blondin and Raymer (2012), with a grid evenly spaced in log($r$), is $\sim 2.5\times$ our standard resolution; these are studies with the highest resolutions so far.

The outer boundary of the domain $r_{\text{out}}$ is fixed at $10.24 R_a$. For the parameters we use, this is large enough to ensure that the flow is always supersonic at the outer boundary, so that the boundary cannot introduce unphysical feedback. We vary the inner boundary $r_{\text{in}}$ between 0.01$R_a$ and 0.04$R_a$ across different simulations, to investigate how different inner boundary size affects the result.

4.1.2 Boundary conditions

We define the upstream (downstream) boundary on the region with $\theta < \pi/2$ ($\theta > \pi/2$). The wind profile we impose on the upstream boundary assumes that the wind follows ballistic trajectories from infinity (see Bisnovatyi-Kogan et al. 1979). i.e. the gravity of the accretor is considered, but the effect of pressure gradient outside the domain is ignored. This allows analytic calculation of the flow properties on the outer boundary. It has been shown that for $M$ as low as 1.4, the supersonic upstream flow can still be well approximated by this ballistic wind profile (Koide et al., 1991). This ballistic wind is also used as the initial condition of the simulation. For the downstream ($\theta > \pi/2$) outer boundary, we apply free flow boundary condition.

For the inner boundary, we apply outflow boundary condition for most of the simulations. We also perform two simulations with an absorbing inner boundary condition for comparison in Section 4.2.3.

4.2 Results

We study whether the stability of the system is affected by Mach number, inner boundary size and resolution. We use two Mach numbers, $M = 3$ and $10$; for each Mach number we use three different inner boundary sizes, $r_{\text{in}} = 0.04, 0.02$ and $0.01 R_a$; then for each pair of ($M, r_{\text{in}}$) we use three different resolutions, which are $1\times, 2\times$ and $4\times$ lowest resolution. This gives a set of 18 simulations in total.

4.2.1 Properties of the flow

For all simulations, the flow is mostly (but not exactly; see §4.2.2) laminar and quickly settles into a steady-state. The flow converges to this steady-state in $\lesssim 10 t_a$ if the simulation is initialized with the ballistic wind profile, and in $\lesssim 2 t_a$ if the simulation is initialized with the steady-state of another simulation with the same $M$. We run each simulation for $20 t_a$, thus for more than half of the time the system is in steady-state.

The density, velocity and Mach number of the steady-state accretion flow for $M = 3$ and $10$, $r_{\text{in}} = 0.01 R_a$ and $4\times$ resolution (our smallest $r_{\text{in}}$ and highest resolution) are shown in Figure 1 and 2 respectively. The flow geometry for the two different Mach numbers are similar, with the shock front slightly closer to the accretor and the shock cone narrower for higher Mach number. Perturbations with wavelength comparable to the cell size are visible in the Mach number profile. Such perturbations are unphysical and are due to misalignment between the grid and the shock (“stair-stepping”); its effect tends to weaken as resolution increases.

The distribution of accretion rate and shock standoff distance for $M = 3$ simulations are summarized in Figure 3. The results for $M = 10$ are nearly identical and are not shown. To avoid the result being affected by the initial condition, the first $10 t_a$ of each simulation is excluded from this figure. Accretion is overall stable, with the amplitude of the accretion rate fluctuation and shock front oscillation less than a few percent for all simulations. Figure 3 shows good numerical convergence with respect to increasing resolution. In general, the amplitude of accretion rate fluctuation and shock front oscillation remain similar or decrease as resolution increases, showing that there is no unresolved instability even at our lowest resolution; and that the unphysical perturbation due to finite grid size is reduced as resolution increases.

4.2.2 Vortex generation from unphysical perturbations

Although the accretion rate is nearly constant for all simulations, the accretion flow is not exactly laminar when the accretor size is small and resolution high. Instead, we observe vortices intermittently generated in front of the accretor for $M = 3$, $r_{\text{in}} = 0.01$ and $M = 10$, $r_{\text{in}} = 0.02$ at $4\times$ resolution, and for 6The real systems we are interested in all have $M \gtrsim 10$. Here we perform $M = 3$ simulations mainly for comparison with previous studies.

7Unless the instability can only be resolved for even higher (> $4\times$) resolution.
Figure 1: A snapshot of the accretion flow for axisymmetric accretion with $M = 3$, $r_{in} = 0.01R_a$ and 4x resolution at $11t_a$. Top left: density and velocity streamlines on large scale. Top right: density and velocity vectors (showing only direction but not amplitude) near the accretor. Bottom left: Mach number on large scale, with the sonic surfaces ($M = 1$) marked in grey. Bottom right: same as bottom left, but near the accretor. The inner sonic surface is attached to the inner boundary. The flow is stable, with some small perturbation due to grid effect.

Figure 2: Same as Figure 1 but for $M = 10$ at $15t_a$. 
Figure 3: The distribution of mass accretion rate (left panel) and shock standoff distance (right panel) for 2D axisymmetric simulations at $\mathcal{M} = 3$ with different inner boundary sizes and resolutions. The fluctuation of accretion rate and oscillation of shock front remain similar or decrease as resolution increases, showing good numerical convergence. The result for $\mathcal{M} = 10$ is very similar.

Figure 4: Snapshot of vorticity profile for axisymmetric accretion with $\mathcal{M} = 3$, $r_{in} = 0.01R_a$ and $4 \times$ resolution, taken at $15.5t_a$. Vorticity is scaled by $r^{3/2}$ to account for the larger velocity and smaller length scale when the flow is closer to the accretor. The vortex on the upstream direction at $\sim 1.5R_a$ is the largest vortex we see in this simulation. Vortices appear only intermittently, and for most of the time, the accretion flow does not host any vortex and is similar to the snapshot shown in Figure 1. Despite perturbation from vortices, the accretion rate and shock standoff distance remains nearly constant. (Unphysical) vorticity perturbation behind the shock due to stair-stepping is also visible; note that the large vorticity on the shock is not due to stair-stepping but corresponds to the divergence of vorticity when there is a discontinuity. See text for more discussion.
Figure 5: Same as Figure 4, but uses mesh refinement to double the resolution near the shock. (Data have been cast to unrefined resolution when making this plot.) Vortex is no longer generated in front of the accretor, and the vorticity perturbation generated by stair-stepping is weaker compared to Figure 4.

$M = 10$, $r_{in} = 0.01$ at $2\times$ and $4\times$ resolution. An example of the vortex generated is shown in Figure 4. Such vortex originates from the front side of the accretor, moves forward (i.e. towards upstream direction) while growing larger, then moves back downstream and is eventually absorbed by the accretor. The typical lifetime of each vortex is $\lesssim 1t_a$. Although the vortices seem to perturb the flow significantly, it barely affects the accretion rate. This is because most of accretion happens behind the accretor (near $\theta = \pi$), and the flow there remains unperturbed as vortices only appear in front of the accretor. In addition, since vortices appear in a region of low flow velocity (this is visible in the Mach number profile in Figure 1 and 2), it has little effect on the location of the shock front. Although the shock front moves back and forth in response to the vortex, the amplitude of such oscillation is at most a few percent of the shock standoff distance (see right panels of Figure 3).

Even though they barely affect accretion rate, the origin of the vortices in axisymmetric BHL accretion is an interesting problem. One possibility is that vortex generation in front of the accretor is related to stair-stepping at the shock front. When the shock front is not aligned with the grid, stair-stepping generates perturbation behind the shock front, which is then advected towards the accretor. Figure 4 clearly shows the generation and advection of vorticity perturbation behind the shock. Such perturbation can sometime make a small portion of the flow miss the accretor; this small portion of overshoot flow creates a weak outflow in front of the accretor, which becomes a vortex upon encountering the incoming flow from upstream. This mechanism can also be interpreted using vorticity conservation: Vorticity perturbation that is generated by stair-stepping and advected towards the accretor but fails to be accreted can be accumulated in front of the accretor, since in this asymmetric flow $\omega/(r \sin \theta)$ is approximately conserved.$^8$

This explanation is consistent with the observation that vortex generation happens only for flow with small $r_{in}$, which makes the flow (and the advecting vorticity perturbation) easier to miss accretor, and high resolution, which corresponds to lower numerical viscosity (vorticity generation by stair-stepping, on the other hand, is not significantly reduced when resolution increases, since the characteristic wavenumber also increases).

To test our explanation, we perform a simulation with $M = 3$, $r_{in} = 0.01$ and $4\times$ (base) resolution with a mesh refinement that doubles the resolution near the shock. Although the increased resolution may not directly reduce vorticity production by stair-stepping, the perturbation generated by stair-stepping should be damped as it passes the refinement boundary due to the sudden drop of resolution. Therefore, vortex generation should be suppressed compared to the simulation without mesh refinement. This is indeed the case; in the simulation with mesh refinement, we never observe vortex generation, and the vorticity perturbation (shown in Figure 5) due to stair-stepping is slightly smaller near the accretor compared to Figure 4. This confirms that the vortices we observe originate from numerical artifacts.

$^8$Gravity, as a conservative force, does not change vorticity. However, pressure gradient that is misaligned with density gradient (since entropy gradient in the flow makes it non-barotropic) can break vorticity conservation. This makes the vorticity conservation argument less rigorous.
Although the perturbations that generate the vorticies are unphysical, this vortex generation mechanism demonstrates that when $r_{in}$ is small, even small perturbation can significantly affect the flow structure in front of the accretor. This is related to why the flow is prone to instability in 3D at small accretor size and finite upstream gradient (see §5).

### 4.2.3 Effect of different boundary condition

From Figure 1 and 2, we observe that the sonic surface is in contact with the inner boundary, i.e. part of the inner boundary is subsonic. [For $\gamma = 5/3$, this has to be the case if the flow is in steady state, as proved by Foglizzo and Ruffert (1997).] Therefore, choosing a different inner boundary condition may affect the feedback at the inner boundary. To test whether this affects the stability of the flow, we redo two of our simulations ($r_{in} = 0.01$, $4\times$ resolution, $M = 3$ and 10) with an absorbing inner boundary condition. Figure 6 compares the distributions of accretion rate and shock standoff distance for different boundary conditions. For $M = 3$, absorbing boundary condition produces significantly less fluctuation because vortices are barely generated in this case. Meanwhile, for $M = 10$, the distributions of accretion rate and shock standoff distance, as well as the properties of the flow, are mostly independent of the boundary condition. Therefore, changing the boundary condition does not significantly affect the properties of the flow, although an absorbing inner boundary condition tends to stabilize the flow more (by pulling the flow adjacent to the inner boundary more strongly).

### 4.2.4 Necessity of the H-correction

Near $\theta = 0$, the shock is approximately aligned with the grid, making the flow prone to numerical artifacts (Quirk, 1994). We suppress such artifacts by introducing extra dissipation near the shock through H-correction (Stone et al., 2008). To illustrate the necessity of this H-correction, we perform a simulation with $M = 3$, $r_{in} = 0.01$ and $4\times$ resolution without the H-correction. Figure 7 shows that removing the H-correction leads to unphysical vortex generation at the shock front; a series of vortices are generated at the shock near $\theta = 0$, and advect with the flow, eventually reaching the accretor. (This is very different from the vortex in Figure 4, which originates near the accretor.) The vorticity perturbation due to stair-stepping at the shock is also stronger compared to Figure 4 (this is most visible in the upper left part of the figure). The H-correction is therefore necessary, since otherwise numerical artifacts can significantly affect flow properties.
4.3 Comparison with previous studies

We find the axisymmetric accretion flow to be overall stable, with nearly constant accretion rate and shock standoff distance; this is in agreement with previous studies (e.g. Pogorelov et al. 2000).

In some simulations, we observe intermittent vortex generation in front of the accretor, which is triggered by unphysical perturbation due to stair-stepping at the shock. Most previous 2D axisymmetric simulations, which have relatively large \( r_{\text{in}} \) and relatively low resolution, do not produce such vortices; this is consistent with our observation that vortex generation only happens at small \( r_{\text{in}} \) and high resolution (low numerical viscosity). The only exception is Koide et al. (1991), which observes a relatively large (compared to ours) vortex for \( r_{\text{in}} = 0.015R_a \) and \( \mathcal{M} = 10 \) (see their Figure 12). Their resolution is lower than our lowest resolution, suggesting that their vortex scheme is either significantly less dissipative or more prone to numerical artifacts compared to ours.

While all our simulations exhibit stable flow, observations of vortex generation suggest that the flow is more sensitive to perturbation when \( \mathcal{M} \) is large or \( r_{\text{in}} \) is small. This \( r_{\text{in}} \) dependence qualitatively agrees with the 3D simulations of Blondin and Raymer (2012). However, Blondin & Raymer report a “breathing-mode” which leads to a quasi-periodic variation of the accretion rate with amplitude \( \sim 10\% \) at \( \mathcal{M} = 3 \), \( r_{\text{in}} = 0.01 \); similar behavior is never observed in our simulations. This breathing mode is likely a 3D effect: Although Blondin & Raymer observe the flow to be highly axisymmetric, they also comment that this is not the case near the accretor for small accretor size, and significant asymmetry can be observed in the mass flux across the inner boundary shown on the lower panel of their Figure 5. Since they also report a very low angular momentum accretion rate (the mean specific angular momentum of accreted material is always \(< 5\%\) of the Keplerian specific angular momentum at \( r_{\text{in}} \)), we suspect that the breathing mode they observe originates from a reflection-symmetric but non-axisymmetric perturbation near the accretor. Additionally, it is possible that this breathing mode is affected by, if not completely due to, the grid geometry, since the Yin-Yang grid they adopt can produce grid effects with reflection symmetry and the high-velocity flow near the accretor is prone to such perturbation (which, for instance, can make part of the flow miss the accretor at certain \( \phi \)).

5 3D BHL accretion with zero or small transverse upstream gradient

Simulations of BHL accretion in 3D with or without an imposed transverse upstream gradient also have a long history (see a review in Foglizzo et al. 2005, which also includes simulations with \( \gamma \) other than 5/3). The earliest works (e.g. Livio et al. 1986; Matsuda et al. 1991) used low resolution and did not resolve the accretor (i.e. diameter of accretor is only 1 - 2 cells). Sawada et al. (1989) is perhaps the first

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9Koide et al. solves the finite volume problem using Osher upwind scheme, while we use a second-order Godunov scheme.
to resolve the accretor (with \( r_{\text{in}} \approx 0.1 R_a \)) thanks to the use of a special radial grid with \( \Delta r/r \) increasing near the accretor, and they find the flow to be stable when there is no upstream gradient and quasi-stable when there is small velocity gradient. A series of works by Ruffert (Ruffert and Arnett, 1994; Ruffert, 1994; Ruffert and Anzer, 1995; Ruffert, 1997, 1999) using a nested Cartesian grid manage to achieve a resolution of \( r/\delta \approx 5 \) (where \( \delta \) is the cell size) for most simulations and \( \sim 10 \) for a few high-resolution simulations, allowing the accretor to be resolved. They find the flow to be always unstable, for \( r_{\text{in}} \) as large as 0.1\( R_a \), regardless of whether there is any upstream gradient. In all simulations, they randomly perturb the initial density of each cell by 3\% in order to break the symmetry of the grid and initial condition. We will show that when there is no upstream gradient, this relatively large initial perturbation is the main reason for the instabilities that observe (see the end of Section 5.2).

More recently, this problem is revisited by Blondin and Raymer (2012) and MacLeod and Ramirez-Ruiz (2015) using higher resolution simulations, with a focus on how the stability of the flow depends on \( r_{\text{in}} \). Blondin and Raymer (2012) simulate BHL accretion with no upstream gradient, and report a stable flow for \( r_{\text{in}} = 0.05 R_a \) and an unstable flow with a near-axisymmetric breathing mode for \( r_{\text{in}} = 0.01 R_a \). (We discussed this breathing mode previously in Section 4.3.) MacLeod and Ramirez-Ruiz (2015) study BHL accretion with finite upstream gradient in the context of accretion within a common envelope and mainly covers the regime of relatively low Mach number (\( M \leq 3 \)) and relatively large density gradient (\( \epsilon_p = 0.1 - 5 \)). They find the flow to be unstable for all simulations with finite \( \epsilon_p \). Comparing results for \( r_{\text{in}} = 0.05 R_a \) and 0.01\( R_a \), they also find that the flow becomes more unstable for smaller \( r_{\text{in}} \).

In this section, we consider high-\( M \) BHL accretion in full 3D, with small (\( \epsilon_p \lesssim 0.1 \)) but finite transverse upstream gradient, which is relevant for many SgXB systems (Table 2). Our simulations follow a formalism largely similar to that of MacLeod and Ramirez-Ruiz (2015), but we cover a different parameter space. We also use multiple \( r_{\text{in}} \) (between 0.005 and 0.04) to thoroughly investigate the \( r_{\text{in}} \) dependence.

### 5.1 Setup

The upstream flow is now allowed to have some finite transverse gradient. We parametrize the transverse gradient such that wind at infinity has

\[
\rho = \rho_\infty \exp(\epsilon_p y), \quad v = v_\infty \exp(\epsilon_v y),
\]

Here \( y \) is a direction normal to the direction of velocity (which we define as \(-\hat{x}\)). The sound speed is set such that the flow is isentropic at infinity, with its value at \( y = 0 \) specified by the Mach number \( M \). We assume that \( \epsilon_p, \epsilon_v \) are both small, so the parameterization (14) is approximately a linear dependence on \( y \).

MacLeod and Ramirez-Ruiz (2015) have used the same setup to study BHL accretion with relatively large upstream density gradient (\( \epsilon_p \) between 0.1 and 5). However, in this section we only consider the case when \( \epsilon_p, \epsilon_v \) are small (\( \lesssim 0.1 \)) or zero, for two reasons: First, many observed systems show small transverse upstream gradient, with \( \epsilon_p, \epsilon_v \lesssim 0.1 \), and it is important to study if the behavior at small gradient is different from that at larger gradient; especially, we want to see whether a realistic \( r_{\text{in}} \) (e.g. \( r_{\text{in}} \sim R_{\text{mag}} \)) can be sufficiently large to make the flow stable when \( \epsilon_p, \epsilon_v \) are small. Second, when transverse upstream gradient is large, a simple parametrization such as (14) may not be a good approximation of the actual wind profile; the effect of companion gravity and orbital motion are also often non-negligible, since the large upstream gradient is usually due to a large \( R_a \). Later in this paper (Section 6.3), we will study systems with larger transverse upstream gradient with an example that adopts a more realistic wind profile with parameters resembling OAO 1657-415.

A list of 3D simulations we perform (which also includes simulations in Section 6), is given in Table 3.

### 5.1.1 Grid and resolution

Since the flow is no longer axisymmetric, we need to switch to a 3D grid. A spherical-polar grid has the symmetry we desire, but to avoid problems near the pole, we adopt a nested cartesian grid.\(^{10}\)

The domain has size 14\( R_a \times 8 R_a \times 8 R_a \), with range \([-10 R_a, 4 R_a]\) for \( x \) and \([-4 R_a, 4 R_a]\) for \( y, z \). Here the coordinate is defined such that the wind comes from \( x \to \infty \), with initial velocity in \(-\hat{x}\) direction. This domain is large enough to ensure that no boundary can introduce unphysical feedback.

\(^{10}\)There are other ways to exploit the advantages of spherical-polar grid while avoiding some of its disadvantages. For instance, the Yin-Yang grid adopted in the 3D simulation of Blondin & Raymer (2012), or polar-averaging which we discuss in Appendix ??.
We also define a spherical-polar coordinate \((r, \theta, \phi)\) centered at the NS to facilitate our later discussions. This coordinate is oriented such that \(\theta = 0\) points to \(+\hat{z}\). Note that this orientation is different from that used in Section 4.

Our nested grid has increased resolution at smaller \(r\) so that the angular resolution remains roughly constant for \(r \lesssim 1R_a\). The default root resolution is 8 cells per \(R_a\), and we refine the mesh at smaller \(r\) by multiple levels (each refinement level increases the resolution by a factor of two) such that \(r/\delta\) (with \(\delta\) being the cell size) is never below 10. This gives \(10 \lesssim r/\delta \lesssim 27\),\(^{11}\) which is comparable to the lowest resolution for our 2D simulations \((r/\delta \approx 20, 1/\delta_0 \approx 30)\). The accretor is well resolved, with a diameter of \(> 20\) cells. Our resolution is similar to that in MacLeod and Ramirez-Ruiz (2015), and lower than that of Blondin and Raymer (2012).

For a few simulations, we modify the domain size and resolution to test convergence, avoid grid effect or save computational cost, and such modifications will be individually introduced when discussing those simulations.

### 5.1.2 Boundary conditions

We implement the upstream boundary condition for the \(+x\) boundary, with the imposed wind profile given by ballistic trajectories from infinity. This is also the default initial condition of the simulation. The downstream region includes all other outer boundaries, and for them we use a free flow boundary condition. For the inner boundary, we use absorbing boundary condition. Physically, this represents accretion with no feedback on the surrounding flow. We have also shown in our 2D simulations (see Section 4.2.3) that the boundary condition from absorbing to outflow barely affects the result.

To avoid physically unstable modes being suppressed by the symmetry of the grid, we introduce a random initial perturbation in each cell with amplitude \(\delta \rho/\rho = 10^{-4}\). We choose this value so that this random perturbation is always much smaller than the effect of upstream density gradient (with the smallest gradient we use being \(\epsilon_{\rho} = 0.01\)). Here we do not attempt to use this random density perturbation to model any physical perturbation (e.g. clumps) in the wind; we discuss the effect of a perturbed clumpy wind in Section 7.4.1.

### 5.2 Results: no transverse upstream gradient

For accretion with no transverse upstream gradient \((\epsilon_{\rho}, \epsilon_v = 0)\), the results of 3D simulations are largely similar to that of axisymmetric 2D simulations. A snapshot of the flow for simulation AS3 is shown in Figure 8. The flow is overall stable (with some minor perturbation due to finite grid spacing at the shock), and the accretion rate is approximately constant. This is also the case for larger accretor size (AS1, AS2). The flow does not show any vortex, which is reasonable given the relatively low resolution (compared to our 2D simulations).

As shown in the right panels in Figure 8, the flow remains largely axisymmetric, and deviation from exact axisymmetry is small. One caveat is that the deviation from exact axisymmetry due to grid geometry can sometimes nontrivially distort the inner acoustic surface (see the bottom right panel of Figure 8; the inner sonic surface is the black contour attached to the inner boundary). This may affect the stability of acoustic modes in the subsonic region. Nevertheless, when the result is less sensitive to small deviation from symmetry (e.g. when the flow is turbulent, as is the case for many simulations below), the cartesian grid should still produce reliable results.

Our result is in agreement with the \(\epsilon_{\rho} = 0\) simulations of MacLeod and Ramirez-Ruiz (2015), which is at a lower Mach number \((M = 2)\). It is also consistent with Blondin and Raymer (2012) in that the flow remains axisymmetric, although we do not reproduce the instability they observe for \(r_n = 0.01\), probably due to our grid geometry or lower resolution. However, this is different from the result of Ruffert and Arnett (1994). Their simulations, despite having lower resolution and larger accretor size (both tend to make the flow more stable), show an unstable flow. This is likely because they use a much larger initial perturbation \((\delta \rho/\rho = 0.03)\) to break the symmetry. We try to reproduce their results by running our simulation with their resolution and \(r_n\), and found that the flow is unstable for \(\delta \rho/\rho = 0.03\) and stable for \(\delta \rho/\rho = 10^{-4}\). Physically, this suggests that the accretion flow can become unstable when the upstream wind contains random perturbation at small length scale (e.g. small clumps in the wind) with sufficiently large amplitude.

\(^{11}\)This is for regions near the accretor; further from the accretor where the resolution is just the root resolution, \(r/\delta\) can be larger.
<table>
<thead>
<tr>
<th>Name</th>
<th>$M$</th>
<th>$r_{in}$</th>
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<th>root resolution</th>
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Table 3: Parameters for 3D numerical simulations. We use an cartesian grid with static mesh refinement, choosing the refinement level at each location so that the local resolution $r/\delta$ is no smaller than 10 (20 for R and VF1R) for $r \geq r_{in}$. For the last five simulations, the initial and boundary conditions are implemented using our parametrized wind model (see Section 2.2) with parameters drawn from corresponding systems (see Table 1 and 2). All simulations (except the resolution studies R and VF1R) run for $\sim 40 \tau_{a}$. 

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5.3 Results: small transverse upstream gradient

For small transverse upstream gradient, we focus on the \( r_{in} \) dependence of stability, and how the strength of \( \epsilon_\rho \) affects \( r_{in} \) dependence. We also investigate whether increasing the Mach number and changing density gradient to a velocity gradient affects the result. In this subsection, we focus on summarizing the simulation results; discussion of relevant physical mechanisms will be given in the next subsection.

5.3.1 \( r_{in} \) dependence

In simulation B1 - B4, we fix the upstream gradient at \( \epsilon_\rho = 0.1 \) and vary \( r_{in} \) from 0.04 to 0.005 to investigate the \( r_{in} \) dependence of accretion. The mass accretion rate \( \dot{M} \) and mean (averaged spatially over the inner boundary but not temporally) specific angular momentum in \( z \) direction of accreted material \( \langle L_z \rangle \) are given in Figure 9. For all four simulations, the accretion flow is unstable, but there is neither visible periodicity nor visible peak in the power spectrum of \( \dot{M} \) or \( \langle L_z \rangle \). Figure 10 shows the distribution of \( \dot{M} \) and \( \langle L_z \rangle / L_{Kep} \) (where \( L_{Kep} \equiv \sqrt{GM NS r_{in}} \) is the Keplerian specific angular momentum at \( r_{in} \)). As \( r_{in} \) decreases, \( \dot{M} \) decreases and the distributions of \( \dot{M} \) and \( \langle L_z \rangle / L_{Kep} \) both widen, suggesting an increase of instability. In addition, the centroid of \( \langle L_z \rangle / L_{Kep} \) becomes closer to zero, so the NS accretes less angular momentum. These behaviors are in broad agreement with the observation of MacLeod and Ramirez-Ruiz (2015) that the flow is more unstable for smaller \( r_{in} \).

Snapshots of the accretion flow, shown in Figure 11, confirm the trend of increasing instability for decreasing \( r_{in} \). For B1 (largest accretor), the flow is only weakly unstable, and is often near-laminar. For B2 - B4, the flow is turbulent everywhere near the accretor. The amount of turbulence kinetic energy increases as \( r_{in} \) decreases, as suggested by the overall increase of distance between the shock and the accretor. The turbulent nature of the flow explains the randomness of \( \langle L_z \rangle \) and the lack of periodicity when \( r_{in} \) is small.

5.3.2 \( \epsilon_\rho \) dependence

In addition to simulations B1 - B4 in which we fix \( \epsilon_\rho \) while varying \( r_{in} \), we also perform simulations at different \( \epsilon_\rho \) (C1 - E4). These simulations illustrate how the stability of the flow depends on \( \epsilon_\rho \).

For all \( \epsilon_\rho \) we used, the flow becomes unstable at sufficiently small \( r_{in} \), and the flow is in general more
Figure 9: Mass accretion rate $\dot{M}$ and mean specific angular momentum in $z$ direction of accreted material $\langle L_z \rangle$ (normalized by $L_{\text{Kep}} \equiv \sqrt{GM_{\text{NS}}r_{\text{in}}}$, which is different for each simulation) for simulations B1 - B4 (same $\epsilon_\rho$, decreasing $r_{\text{in}}$). As $r_{\text{in}}$ decreases, the flow becomes more unstable, with accretion rate decreasing and the amplitude of $\langle L_z \rangle/L_{\text{Kep}}$ fluctuation increasing, which eventually becomes comparable to $L_{\text{Kep}}$. $\dot{M}$ and $\langle L_z \rangle$ show no visible periodicity.

Figure 10: Distribution of $\dot{M}$ and $\langle L_z \rangle/L_{\text{Kep}}$ for simulations B1 - B4. As $r_{\text{in}}$ decreases, $\dot{M}$ decreases and the distributions of $\dot{M}$ and $\langle L_z \rangle/L_{\text{Kep}}$ widen. Note that the amplitude of $\langle L_z \rangle$ fluctuation does not necessarily increase, since $L_{\text{Kep}}$ decreases for decreasing $r_{\text{in}}$. 
Figure 11: Snapshots of accretion flow for B1 - B4. Each row gives snapshot of one simulation, sliced at $z = 0$. From left to right: Density ($\rho/\rho_\infty$); Mach number; azimuthal velocity $v_\phi/v_{Kep}$ where $v_{Kep} \equiv \sqrt{GM_{NS}/r}$ and $r, \phi$ are defined for the spherical-polar coordinate with the north pole ($\theta = 0$) in +\hat{z} direction; $v_r/v_{ff}$ with the free-fall velocity $v_{ff} \equiv \sqrt{2GM_{NS}/r}$; radial mass flux per unit solid angle, defined as $d\dot{M}/d\Omega = r^2 \rho v_r$, and normalized by $\dot{M}_{HL}/4\pi$. As $r_{in}$ decreases, the flow becomes more unstable and eventually turbulent.
Figure 12: Same as Figure 9, but for simulations A1 - E1 (same $r_{in}$, different $\epsilon$ values). The accretion rates for A1, D1, E1 are near identical.

Figure 13: Same as Figure 9, but for simulations D1 - D4.
Figure 14: Same as Figure 9, but for simulations E1 - E4.

Figure 15: Same as Figure 10, but for unstable simulations at $r_{in} = 0.01$ (B3, D3) and $r_{in} = 0.005$ (B4, D4, E4). Distributions of $\dot{M}$ is sensitive to $\epsilon_p$, but show little dependence on $r_{in}$ at given $\epsilon_p$. 
prone to instability at larger $\epsilon_\rho$. In Figure 12, we compare simulations B1 - E1, which have the same $r_{in} = 0.04$ and decreasing $\epsilon$. For B1 and C1 ($\epsilon_\rho = 0.1$ and 0.05), the flow is unstable, with C1 having $\dot{M}$ fluctuation with smaller amplitude; for D1 and E1 ($\epsilon_\rho = 0.02$ and 0.01), the flow is stable, with accretion rate similar to the zero transverse gradient case (AS1). This shows that at given $r_{in}$, the flow is more stable for smaller $\epsilon_\rho$. We also find that the value of $r_{in}$ below which the flow becomes unstable decreases as $\epsilon_\rho$ decreases. For $\epsilon_\rho = 0.1$ and 0.05, the flow is already unstable at $r_{in} = 0.04$; for $\epsilon_\rho = 0.02$ and 0.01 (Figures 13 and 14), the flow becomes unstable at $r_{in} = 0.01$ and 0.005 respectively. We will further discuss the criterion of instability (which depends on both $\epsilon$ and $r_{in}$) in Section 5.4.3.

Another interesting feature is that for sufficiently small $r_{in}$, the distribution of $\dot{M}$ and $\langle L_z \rangle$, although sensitive to $r_{in}$, show little (if any) dependence on $\epsilon_\rho$, as shown in the comparison in Figure 15. This result appears somewhat unexpected, but it is reasonable given that for small $r_{in}$ the flow near the accretor is highly turbulent and should not be sensitive to small gradients in the upstream flow. Note that this may no longer be the case when $\epsilon_\rho$ is large; in that case the accretion flow contains a large angular momentum which can force the formation of disk-like structures even when the flow is turbulent. An example of this is simulation OAO, which we discuss in Section 6.3.

### 5.3.3 Effect of higher Mach number and velocity gradient

So far, we only discussed the behavior at a fixed Mach number ($\mathcal{M} = 10$) for finite upstream density gradient ($\epsilon_\rho \neq 0$, $\epsilon_\upsilon = 0$). Here we use two examples (simulations M and V) to briefly discuss the effect of higher Mach number and velocity gradient.

The distributions of $\dot{M}$ and $\langle L_z \rangle$ for B3, M and V are shown in Figure 16. For simulation M, parameters are identical to B3 except $\mathcal{M}$ is increased to 30. Increasing the Mach number barely affects the behavior of the flow near the accretor and the distributions of $\dot{M}$ and $\langle L_z \rangle$. Therefore, our simulations at $\mathcal{M} = 10$ should be applicable to real SgXB systems, although most of them have $\mathcal{M} > 10$ (see Table 2).

For simulation V, parameters are identical to B3 except density gradient $\epsilon_\rho = 0.1$ is changed to velocity gradient $\epsilon_\upsilon = 0.1$. The result for V is qualitatively similar to B3, but the distribution of $\dot{M}$ is slightly wider, suggesting more instability. A perhaps more interesting result is that the distribution of $L_z$ is now clearly centered at a negative value, unlike the case for finite density gradient. This is because the flow from $y > 0$, with more velocity, requires a smaller impact parameter to be accreted (note that $R_a \propto v_\infty^{-2}$). Although the accreted material from $y > 0$ contains more specific angular momentum, more mass is accreted from $y < 0$. When the latter effect overpowers the former, $\langle L_z \rangle$ tends to be negative.\(^{12}\)

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\(^{12}\)This analysis assumes a laminar accretion flow, and it is not obvious a priori whether the same argument holds when the
Our observation of a preferentially negative $\langle L_z \rangle$ is consistent with previous works with finite $\epsilon_\rho$, such as Ruffert and Anzer (1995) and Ruffert (1997).

5.4 Physical explanation of flow morphology

5.4.1 Flow morphology

To study the physical origin of the instability, we first discuss the morphology of the flow for our simulations. Here we consider B1 - E4, which have identical parameters and setups except $r_{in}$ and $\epsilon_\rho$. Overall, the flow pattern observed in these simulations can be classified into three types: (a) Stable flow (D1, D2, E1 - E3), with negligible variation of $\dot{M}$ and $\langle L_z \rangle$. For stable flow, $\langle L_z \rangle$ always aligns with $\epsilon_\rho$ (i.e. they have the same sign). (b) Weakly unstable flow (B1, C1), characterized by relatively small variation of $\dot{M}$ and $\langle L_z \rangle$ (typically, $\dot{M}$ fluctuation amplitude is $\lesssim 20\%$), and $\langle L_z \rangle$ has a strong preference to align with $\epsilon_\rho$. In this case, the flow is in general unstable, but not always turbulent; especially, the flow behind the accretor is mostly laminar (e.g. see top panels in Figure 11). (c) Highly unstable and turbulent flow (B2 - B4, D3, D4, E4), characterized by an always turbulent flow near the accretor; variation of $\dot{M}$ and $\langle L_z \rangle$ are both large, and $\langle L_z \rangle$ no longer show a significant preference to align with $\epsilon_\rho$.

5.4.2 Mechanism of instability

The distinction between the three regimes can be analyzed by considering the angular momentum of the accretion flow. Consider, for instance, $\epsilon_\rho > 0$, $\epsilon_\phi = 0$. Since the flow initially coming from $y > 0$ has larger density, when the flow from $y > 0$ and $y < 0$ meet behind the accretor the resulting flow should have a positive $L_z$. Of course, $L_z$ is not conserved as the flow goes towards the accretor, but it is reasonable to assume that there is no order-of-magnitude change in $L_z$ if the flow is laminar.\(^{13}\) When $L_z \lesssim L_{Kep}$ for all material that will eventually be accreted, all material can be directly accreted through a near-radial accretion flow, leading to a stable flow and a constant $\langle L_z \rangle$, which is $\propto \epsilon_\rho$ for flow with small density gradient. This produces the case of stable accretion flow.

However, when the accretor size becomes smaller, $L_{Kep}$ decreases and part of the flow cannot be directly accreted; instead, it tends to overshoot the accretor, with $v_\phi \gtrsim v_{Kep}$. This can destabilize the flow by forming a strong velocity shear, since the $v_\phi$ of the incoming flow near the accretor is in general small. This is qualitatively similar to the case for 2D axisymmetric simulation where accretion flow that overshoots the accretor (due to unphysically produced perturbation in the flow instead of upstream gradient) creates vortices. However, the 3D geometry allows the flow to be more unstable; usually, the flow in front of the accretor eventually becomes turbulent.

The turbulent flow in front of the accretor tends to expand to the downstream side, and produces two opposing effects. First, the accretion flow that would miss the accretor due to its angular momentum excess can now dispose its angular momentum by colliding with the turbulent flow, which reduces overshooting and suppress instability. Second, when the turbulence is strong enough, it can significantly perturb the flow behind the accretor and increase its transverse velocity, which promotes overshooting and increases instability.

When the first effect is more prominent, the flow is only weakly unstable because the instability tends to self-amplify until the flow becomes fully turbulent everywhere near the accretor. The turbulent nature of the flow near the accretor makes it barely sensitive to the upstream gradient, and $\langle L_z \rangle$ no longer show a clear preference to align with $\epsilon_\rho$. This also explains why the distribution of $\dot{M}$ and $\langle L_z \rangle$ in this regime shows dependence only on $r_{in}$ but not on $\epsilon_\rho$.

To confirm our interpretation, we study the development of instability in D3 and E4. These are examples where the simulation starts with a near-laminar flow and eventually becomes unstable. the

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\(^{13}\)A weak spiral shock (hardly spiral due to the large pitch angle) attached to the accretor can appear (see top panels in Figure 11), leading to a reduction of $L_z$ as the flow crosses the shock. Still, this reduction does not affect the order-of-magnitude result.
Figure 17: Similar to Figure 11, but for simulation E4 at different epochs, showing development of instability. The simulation starts at $t = 120t_a$. At $t = 121t_a$ (first row), flow is still overall stable; at $t = 127t_a$ (second row), the flow in front of the accretor is already turbulent due to the flow behind the accretor (with $L_z \gtrsim L_{Kep}$) overshooting the accretor; at $t = 131t_a$ (third row), the turbulent region expands and the flow is turbulent everywhere near the accretor.
Figure 18: Stability of simulations with finite $\epsilon_\rho$ (B1 - E4). The black dashed line marks the instability threshold $r_{in} = 50\epsilon_\rho^2$ [see Eq. (17)], which agrees with all simulations. Two simulations (D2 and E4) lie on this threshold with different stability, tightly constraining the prefactor if we assume that the critical $r_{in}$ is approximately $\propto \epsilon_\rho^2$.

5.4.3 Threshold of instability
Our physical interpretation of the mechanism of instability allows an estimate of the threshold of instability. The flow should turn unstable when $L_z \gtrsim L_{Kep}$. When flow from $y \sim \pm R_a$ meet behind the accretor, the specific angular momentum of the flow should be (in code unit, assuming only density gradient)

$$L_z \sim \epsilon_\rho.$$  \hfill (15)

Meanwhile, $L_{Kep} = r_{in}^{1/2}$. Therefore, the condition of instability is

$$r_{in} \lesssim \epsilon_\rho^2.$$  \hfill (16)

We can calibrate this scaling using our simulations. The stability of simulations with finite $\epsilon_\rho$ (B1 - E4) are summarized in Figure 18. If we assume that the scaling of the threshold follows (16), D2 and E4 which have opposite stability but the same $r_{in}/\epsilon_\rho^2$ must lie on the instability threshold, and the condition of instability needs to be

$$r_{in} \lesssim 0.005 \left( \frac{\epsilon_\rho}{0.01} \right)^2 = 50 \epsilon_\rho^2.$$  \hfill (17)

This result is consistent with all our simulations in Figure 18. Note that this result should only be applicable for high Mach number and small accretor, i.e. when $r_{in}$ is much smaller than the distance between the shock and the accretor.

In general, for a flow with both density and velocity gradient, we expect

$$r_{in} \lesssim 50(\epsilon_\rho + \alpha \epsilon_v)^2,$$  \hfill (18)

with $\alpha$ being some $O(1)$ constant, and is likely negative since positive $\epsilon_\rho$ produced negative $\langle L_z \rangle$. |$\alpha$| is likely $> 1$, given the more unstable flow shown in simulation V. The instability threshold may also
depend on upstream entropy gradient (which is taken to be 0 in our simulations), but the scaling should remain overall similar.

The factor of 50 in Eq. (17) is rather surprising, since our rough analytic argument and previous analytic estimates (see a review in Ho 1988; note that these estimates are in general not rigorous enough, as Ho pointed out) would both predict this factor to be $O(1)$ (i.e. when $\langle L_z \rangle$ becomes comparable to $L_{Kep}$). In other words, the flow is significantly more prone to instability as one might naively expect. This is because having the maximum (instead of mean) $L_z$ of material going towards the accretor greater than $L_{Kep}$ is sufficient to trigger the instability, and the maximum $L_z$ can be much greater than its mean value, which is indeed $\propto \epsilon_\rho$ with a $O(1)$ prefactor as we show in Section 7.1. One caveat here is that our analysis implicitly assumes the ratio between maximum and mean $L_z$ to be independent of $\epsilon_\rho$, which is not obviously correct.

5.5 Difference between 2D and 3D BHL accretion: flip-flop instability and disk formation

The simulations in this section illustrate the key difference between 2D planar and 3D BHL accretion with zero or small upstream gradient. For 2D planar BHL accretion, the flow exhibits a flip-flop instability, which leads to the overstable oscillation (or wobbling) of the shock cone behind the accretor even when there is no upstream gradient. However, similar to previous studies (Blondin and Raymer, 2012; MacLeod and Ramirez-Ruiz, 2015), we never observe flip-flop instability in 3D: The instability we observe at sufficiently large $\epsilon_\rho$ and sufficiently small $r_{in}$ is due to a completely different mechanism (see 5.4.2), and the downstream flow beyond the stagnation point shows little transverse perturbation.

As a consequence of the different instability mechanisms, 2D planar and 3D simulations also give different results regarding disk formation. For 2D planar simulations, at sufficiently small $r_{in}$ flip-flop instability can produce enough angular momentum to cause the spontaneous formation of a disk around the accretor, and such disk is stable for $\gamma = 5/3$ (Blondin, 2013). In 3D, however, this is no longer possible since the instability no longer involves transverse motion of the shock and cannot increase the angular momentum of the accretion flow. Instead, turbulence produced by instability can effectively reduce the flow’s mean specific angular momentum, inhibiting disk formation. Moreover, the high temperature of the flow near the accretor (partly due to turbulent heating) prevents the formation of a thin disk and makes any disk-like structure less stable. Consistent with this argument, no persistent, rotationally-supported accretion disk has been observed in B1 - E4. In some simulations, we do observe transient, turbulent disk-like structures around the accretor with $|v_0| \sim v_{Kep}$ (e.g. the snapshot for B3 in Figure 11). However, such disk-like structures are unstable and have a very short lifespan ($\lesssim$ a few $t_a$, which is $\lesssim$ several hours for real systems). They should not be described or studied using models for stable, rotationally-supported disks.

It is worth noting that the angular momentum in the accretion flow is relatively small for all simulations in this section. At sufficiently large $\epsilon_\rho$, the accretion flow may contain enough angular momentum to circulate before reaching the accretor; this may produce a persistent disk (or disk-like structure). We will use one numerical example to explore this regime in Section 6.3, and discuss the criterion of disk formation based on analytic arguments in Section 7.2.

6 Application to real systems

In this section, we use a more realistic wind model to investigate accretion in SgXB systems. We perform simulations with parameters resembling two systems in Table 2 that can serve as limiting cases: Vela X-1 with fast wind and OAO 1657-415.

Vela X-1 with fast wind has very small upstream gradient, and the orbital effect is weak ($R_a/R_H$ and $\Omega_b t_a$ small). Although it may not represent the actual Vela X-1 (whose wind speed is likely much lower), it is representative of other systems with high wind speed such as 4U 1907+097. Here we choose Vela X-1 with fast wind as the representative system because this allows a direct comparison of results with previous studies (e.g. Blondin et al. 1990; Manousakis and Walter 2015), which usually assume fast wind speed when simulating Vela X-1, in accordance to early observational results.

OAO 1657-415 has the largest upstream gradient among the systems in Table 2, and the orbital effect is non-negligible. In previous sections, we only considered the case of small or zero upstream gradient; the large upstream gradient ($\epsilon_\rho \sim O(1)$) of OAO 1657-415 may produce qualitatively different behaviors. We will also compare our results with observations of the system, and discuss whether an accretion disk can be formed.
Figure 19: Same as Figure 9, but for simulations VELA1 - VELA3. VELA3 uses the flow of VELA2 at $t = 60$ as the initial condition. The flow is always stable. $\langle L_z \rangle$ for the three simulations are nearly identical.

6.1 Setup

The setup is similar to simulations in the previous section. The main difference is that we now use the single-star wind model given in Section 2.2 for upstream boundary condition and initial condition, and include the gravity of the companion and orbital effects as discussed in Section 3.1. To make the single-star wind profile self-consistent, we introduce additional acceleration and heating (which depend only on location and is constant in time) such that if NS gravity is turned off, the single-star wind profile is a steady state. In the upstream, the additional acceleration and heating mainly correspond to line-driven acceleration of the wind. Behind the shock, the additional acceleration is unphysical, but its effect should be negligible since such acceleration and heating is very small compared to gravity of the NS or pressure gradient. The parameters of the system are chosen to agree with those of Vela X-1 with fast wind (simulations VF1 - VF3) and OAO 1657-415 (simulation OAO), except that we increase the temperature so that $M = 10$ at the NS for single-star wind profile.

The simulation is performed in a rotating frame, with coordinate oriented such that the single-star wind velocity is in $-\hat{x}$ at the NS and the binary orbit lies on the $xy$ plane. In definition of $R_a$ and normalizations, since there is no longer well-defined $\rho_\infty$ and $v_\infty$, we replace them by $\rho_0$, $v_0$ which are $\rho$, $v$ evaluated for the single-star wind profile at the NS.

Simulations VF1 - VF3 have small gradient, so we can use the same domain and resolution as in the previous section. For simulation OAO, since the streamlines are significantly curved in the whole domain (due to orbital motion and companion spin), we double the root resolution to avoid unphysical upstream features caused by grid effects. The resolution near the accretor remains the same. To accommodate the flow, we also change the domain to $[-6, 2] \times [-4, 4] \times [-2, 2]$. Under this choice, the companion is partially inside the domain. We impose the single-star wind profile for $1.02R_c < R < 1.25R_c$ to represent the surface of the companion.
Figure 20: A snapshot of accretion flow for VELA3, similar to Figure 11. The three rows show slices at $z = 0$, $y = 0$ and $x = 0$ respectively. The flow is almost axisymmetric.
6.2 Results: Vela X-1 with fast wind

VF1 - VF3, which represent to Vela X-1 with fast wind at $r_{\text{in}} = 0.01, 0.005$ and 0.0025, all show stable accretion flow, as shown in Figure 19. The accretion rate remains approximately constant, and $\langle L_z \rangle$ undergoes some fluctuation. To verify the origin of the $\langle L_z \rangle$ fluctuation and confirm that the scale of $\langle L_z \rangle$ is not affected by our relatively low resolution (the angular size of the cell, $b/r$, is larger than $\epsilon_\rho$), we continue VF1 with double resolution (simulation VF1R). In VF1R, fluctuation of $\langle L_z \rangle$ is significantly reduced, and the scale of $\langle L_z \rangle$ remains the same, suggesting that the fluctuation is only due to grid effect related to finite resolution and VF1 - VF3 should give $\langle L_z \rangle$ with tolerable error. A snapshot of the flow for VF3 is shown in Figure 20; the flow is almost axisymmetric.

Comparing this result with simulations including only $\epsilon_\rho$, shows that that using a realistic wind profile increases stability. In E4, the flow becomes unstable at $r_{\text{in}} = 0.005$ for $\epsilon_\rho = 0.01$, while here for similar $\epsilon_\rho$ the flow is still stable at $r_{\text{in}} = 0.0025$. The reason of this increased stability is still unclear. One possibility is that the stability is increased by the velocity gradient $\epsilon_v$, which is smaller than but still comparable to $\epsilon_\rho$. However, this does not seem likely since a negative $\epsilon_v$ should further increase the angular momentum in the accretion flow, making it more unstable. It is also possible that stability is increased by the inclusion of orbital dynamical effects, especially the Coriolis force: The relative strength of Coriolis force (compared to NS gravity) at $r \sim R_N$ for $v \sim v_0$ is $\sim \Omega_b t_a$, which is comparable to $\epsilon_\rho$. (Meanwhile, the relative strength of centrifugal force is $\sim (\Omega_b t_a)^2$ and companion gravity $\sim (R_a/R_H)^3$, which are much smaller.)

For Vela X-1 with fast wind, assuming $R_{NS} = 11$ km, $B_0 \sim 10^{12}$ G and $L_\infty = 4 \times 10^{36}$ erg/s (Walter et al., 2015) gives $R_{mag}/R_N \sim 0.0065 > 0.0025$. This result implies that the accretion flow for systems with very high wind speed is likely stable, if angular momentum transport is efficient within the magnetosphere and there is no other upstream perturbation (e.g. clumps in wind; see 7.4).

Our result for Vela X-1 with fast wind is very different from the 2D planar simulations by Blondin et al. (1990) and Manousakis and Walter (2015), both of which use similar stellar and wind parameters but find the flow (and the accretion rate) to be variable. This difference is mainly because these simulations include the radiative acceleration of the wind (and the suppression of which by NS X-ray photoionization). Manousakis and Walter (2015) produce the observed quasi-periodicity and off-states of Vela X-1; the fact that these behaviors are not produced in our 3D hydrodynamic simulations suggests that these behaviors are closely related to radiative acceleration and X-ray photoionization. Additionally, the 2D planar geometry produces strong flip-flop instability, which is not present in our 3D simulations.

6.3 Results: OAO 1657-415

Simulation OAO lies in a very different regime of the parameter space: the upstream gradient is large ($\epsilon_\rho = 0.44$), and companion gravity and orbital effects are very important ($R_a/R_H \sim 1$). It is worth noting that our results for OAO 1657-415 may not be directly generalized to other systems with large upstream gradient, since qualitative behaviors of the flow may depend on the relative strength between different effects.

The $\dot{M}$ and $\langle L_z \rangle$ evolution, their distribution, and a snapshot of the flow are shown in Figure 21, 22, 23 respectively. The flow is highly asymmetric due to large curvature of the upstream wind and small separation between the NS and the companion ($R_a/D = 0.41$). It is also highly turbulent, with stronger shocks around the accretor than in simulations with turbulent flow at small $\epsilon_\rho$. The accretion rate is much lower and the distribution of $\dot{M}$ wider (Figure 22) compared to simulations with smaller $\epsilon_\rho$, agreeing with the trend we observe in the previous section.

One distinctive feature of simulation OAO is that it exhibits a persistent disk-like structure around the accretor: the flow on the x-y plane near the accretor has $v_\phi \sim v_{K,\phi}$. This leads the preferential alignment of $L_z$ with the upstream gradient, which is visible in Figure 9 and 10. The NS is thus allowed to accrete angular momentum efficiently, explaining why OAO 1657-415 has significantly larger spin rate than other systems in Table 1. This disk-like structure, however, is different from a rotationally supported accretion disk from several important aspects. It is highly turbulent, thick and variable, and the disk-like structure does not show any overdensity on the midplane. More importantly, accretion does not happen mainly through this disk-like structure; Instead, a significant amount of accretion happens near the poles, as is visible in Figure 24.

Our result is consistent with the observations that OAO 1657-415 undergo steady spin-up (Jenke et al., 2012)\textsuperscript{14} and that its accretion rate is inconsistent with BHL-like (i.e. $\dot{M} \sim \dot{M}_{\text{BHL}}$, corresponding

\textsuperscript{14}Jenke et al. (2012) also observe a mode where the NS spins down at a rate uncorrelated with the flux. This may correspond to occasional disruption of the disk-like structure (possibly due to physical effects that we do not include, such as radiative
Figure 21: Same as Figure 9, but for simulation OAO.

Figure 22: Same as Figure 10, but for simulation OAO.
Figure 23: Same as Figure 20, but for simulation OAO. The flow is highly turbulent, but shows a persistent disk-like structure.
Figure 24: Similar to Figure 23, but plots the time-averaged flow. Significant accretion happens through polar regions.
to a stable or weakly unstable flow) wind-fed accretion (Taani et al., 2018). Although these observations are usually used to suggest the existence of an accretion disk, our result show that a turbulent disk-like structure (which is neither thin nor rotationally supported) is also consistent with the observations. Note that we do not rule out the possibility of disk formation; in principle, disk formation is still possible provided that cooling is sufficient (see §7.2).

7 Discussion

7.1 Angular momentum of the accretion flow

In our previous discussion, we assume that a laminar accretion flow contains a mean specific angular momentum \( \propto \epsilon \rho \) and independent of \( r_{in} \). We test this assumption by comparing \( \langle L_z \rangle / \epsilon \rho \) for simulations with stable accretion flow at different \( \epsilon \rho \) and \( r_{in} \) in Figure 25. All of them, except D1 and E1 which have \( r_{in} \) comparable to the shock standoff distance, have

\[
\langle L_z \rangle \approx 0.4 \epsilon \rho R_a v_\infty.
\]

(19)

This confirms that \( \langle L_z \rangle \) is proportional to the specific angular momentum supplied by upstream gradient when accretion flow is laminar.

7.2 Formation of disk-like structures

In our simulations, we find that when \( \epsilon \rho \) is small, stable disk-like structure cannot form (§5.5), but when \( \epsilon \rho \) is relatively large, a persistent disk-like structure (which is still not a rotationally supported accretion disk) forms around the accretor (§6.3). Here, we discuss how the formation of disk-like structure depends on the strength of upstream gradient. In the discussion below, “disk” only means a disk-like structure (i.e. \( v_\phi \sim v_{Kep} \)) which may or may not be a rotationally supported (thin) disk.

To maintain a disk with outer radius \( R_d \), the specific angular momentum of the incoming flow should be at least \( L_z / R_a v_\infty \sim \sqrt{R_d / R_a} \). In 3D simulations, this specific angular momentum is mainly due to the upstream gradient, and should be \( L_z / R_a v_\infty \sim \epsilon \rho \) (assuming \( \epsilon \rho \geq \epsilon_v \)). Therefore, having

footnotes:

15Note that for VF1 - VF3, the angular momentum of the accretion flow is also affected by upstream velocity gradient and orbital effects, and their similarity to simulations with only density gradient is coincidental.

16In 2D BHL accretion, specific angular momentum can also be supported by spontaneously developed asymmetry of the shock (even when there is no upstream gradient), which can lead to the development of a disk-like structure (Blondin, 2013). However, similar behavior is never observed in 3D, due to the absence of flip-flop instability.
$R_d/R_a \sim \epsilon_p^2 \gtrsim \rho_{in}/\rho_a$ is a necessary condition of disk (or disk-like structure) formation.

The analysis above assumes the angular momentum budget to be similar to that of a stable accretion flow. However, the angular momentum budget should be much less when $\epsilon_p^2 \lesssim R_{\text{shock}}/R_a$, because the flow around the disk has to be turbulent. (Consider the instability criterion in §5.4.3, but replace $\rho_{in}$ by $R_d$ which is $\lesssim \epsilon_p^2 R_a$.) As we observed in §5.3, this turbulent flow can reduce the mean specific angular momentum in the flow (compared to when the flow is stable). As the angular momentum budget decreases, the disk has to shrink. In fact, it is reasonable to believe that a disk can never develop, since it has been shown in §5.3 that the ratio between $\langle L_z \rangle / L_{\text{Kep}}$ (evaluated at $\rho_{in}$) decreases as $\rho_{in}$ decreases.

On the other hand, when $\epsilon_p^2 \gtrsim R_{\text{shock}}/R_a$, the disk will be in direct contact with the shock, and there is little room in between for turbulence to reduce the angular momentum budget or disrupt the disk. As an example, in simulation OAO, $R_d \sim \epsilon_p^2 R_a \sim R_{\text{shock}}$ and the flow exhibits a persistent disk-like structure in contact with the shock (Figure 23).

Having sufficient angular momentum budget near the accretor guarantees the formation of a disk-like structure; however, this disk-like structure never becomes a rotationally supported disk in our simulations. This is mainly due to the lack of compressibility. For a $\gamma = 5/3$ adiabatic flow, the pressure becomes high near the accretor and the strong (and asymmetric) pressure gradient prevents the formation of a thin, rotationally supported disk. Turbulent shocks in the flow also produces heating, further decreasing compressibility. Previous studies also demonstrated that if the compressibility is increased by changing the equation of state (see Figure 12 of MacLeod and Ramirez-Ruiz 2015) or by cooling (El Mellah et al., 2018a), forming a rotationally supported thin disk becomes possible.

### 7.3 Regimes of the parameter space

Combining our stability criterion at small $\epsilon_p$ and the discussion about disk formation in the previous subsection, we can now divide the parameter space into regimes of different behaviors. Despite the large number of relevant parameters, we will mainly focus on the $\langle \rho_{in}, \rho_{in}/R_a \rangle$ parameter space and assume that other effects (e.g. $\epsilon_v$, orbital effects) are weaker.

The $(\epsilon_p, \rho_{in}/R_a)$ parameter space can be divided into the four regimes; approximate boundaries between these regimes are shown in Figure 26, together with parameters of observed systems and recent simulations. Below, we summarize the morphology of the flow and the mass and angular momentum accretion rate for each case.

1. $\epsilon_p^2 \lesssim \rho_{in}/50R_a$: The flow should be stable, with approximately radial flow near the accretor, and $\dot{M} \sim \dot{M}_{\text{HL}}$, $\langle L_z \rangle \sim 0.4 \epsilon_p R_a v_{\infty}$.
2. $\rho_{in}/50R_a \lesssim \epsilon_p^2 \lesssim \rho_{in}/R_a$: The flow near the accretor is highly turbulent ($\langle L_z \rangle$), and $\dot{M}$ and $\langle L_z \rangle$ undergo large random variation. The variation of $\dot{M}$ and $\langle L_z \rangle$ increase and their mean values decrease as $\rho_{in}$ decreases, but they barely depend on $\epsilon_p$. Disk formation is impossible, due to insufficient angular momentum budget. This and the previous regime have been simulated and discussed in §4.
3. $\rho_{in}/R_a \lesssim \epsilon_p^2 \lesssim R_{\text{shock}}/R_a$: The behavior should be very similar to the previous case: the flow is turbulent, and disk formation is unlikely. Although the upstream flow now provides an angular momentum budget that is in principle sufficient for disk formation, the turbulent flow between the shock and the accretor can very effectively screen this angular momentum supply and prevent disk formation (§7.2). So far, there has been no simulation that lies unambiguously inside this regime (i.e. sufficiently far from both boundaries) since that would require a very small $\rho_{in}$; however, many observed systems do (Figure 26). Therefore, this regime calls for more investigation in future works.
4. $\epsilon_p^2 \gtrsim R_{\text{shock}}/R_a$: Given the large upstream gradient, a disk-like structure that is in contact with the shock is guaranteed to form, which may become a rotationally supported disk if cooling (not included in this study) is efficient (§7.2). Our simulation OAO lies in this regime, and shows a persistent but turbulent disk-like structure with highly variable $\dot{M}$ (due to the turbulence) and variable but mostly positive $\langle L_z \rangle$ whose mean value is comparable to $L_{\text{Kep}}$. The NS should spin up due to efficient angular momentum accretion.

Simulations of BHL accretion with only upstream density gradient ($\epsilon_p$) agree well with this picture. However, when velocity gradient and orbital effects are also included (in attempt to model realistic systems), the stability of the flow seem to be increased (see §6.2, and green points in Figure 26). Especially, one of the simulations in El Mellah et al. (2018a) (the green star in Figure 26) shows stable flow but lies deep inside a regime where a turbulent flow is expected. If this is not due to our inaccurate estimation of $\epsilon_p$ (which is quite possible, since our method of estimating $\epsilon_p$ based on single-star wind profile may not be appropriate in their numerical setup where radiative wind acceleration inside the NS Roche lobe is ignored), it will suggest that the stability of the flow is in fact sensitive to velocity gradient and orbital
Figure 26: Transverse upstream gradient $\epsilon_\rho$ and inner boundary size $r_{in}$ for simulations in this work and other recent studies. Color marks the stability of the flow and the type of the simulation (with only density gradient, or using more realistic setups that also include velocity gradient and orbital effects such as Coriolis force). Marker shape signifies the source of the simulations: Triangles are from this work (B1 - E4, VF1 - 3, OAO), diamonds from MacLeod and Ramirez-Ruiz (2015), and stars from El Mellah et al. (2018a). Estimated $\epsilon_\rho$ and $R_{mag}/R_a$ (assuming $R_{mag} \sim 10^9$ cm) for some observed systems are plotted in light grey crosses; we include all systems in Table 1 and 2 except “Vela X-1 (fast)”, which adopts a less realistic high wind speed. The dashed and dotted lines mark $r_{in} = 50\epsilon_\rho^2$ and $R_{\text{shock}} = \epsilon_\rho^2$ (with $R_{\text{shock}} \approx 0.15$; all lengths are in $R_a$) respectively; these are approximately the boundaries between different parameter regimes (see text).
effects even though they appear weaker than the density gradient. In that case, directly using our above results to predict the stability of the accretion flow in real systems may not be suitable.

7.4 Other factors that may affect accretion dynamics

7.4.1 Perturbations in the line-driven wind

In reality, the stellar wind from the companion is not necessarily smooth and unperturbed. Variability at the windbase (Poe et al., 1990) together with the line-deshadowing instability (Lucy and White, 1980) can make the wind “clumpy”. Observational results are also consistent with a clumpy wind (Surlan et al., 2013). Numerical simulation by Sundqvist et al. (2018) shows that for typical O star parameters, the clumps in the wind can be over one order of magnitude denser, with size $\sim 0.05R_c$ at $\sim 2R_c$ (this is also the typical location of the NS in a SgXB undergoing wind accretion). For systems with fast wind (see Table 2) the typical size of the clumps can be $\sim R_a$; this should significantly affect the accretion flow, making it highly variable and reducing the accretion rate, as is shown in the simulation of El Mellah et al. (2018b).

Still, this does not mean that investigating accretion from a smooth (or slightly perturbed) upstream flow is irrelevant for SgXB systems, since the perturbation in the wind may not be as strong for different stellar parameters, and other physical mechanisms may suppress the initial perturbation that triggers the instability, making the variability small (if not zero) at the NS.

7.4.2 Radiation from NS

Radiation from the NS ionizes the wind, reducing the wind speed near the accretor by suppressing radiative acceleration. This could significantly affect the behavior of the accretion flow. For example, the 2D simulation in Manousakis and Walter (2015) includes the effect of radiative driving (using CAK/Sobolev model) and its suppression by NS radiation (using the $\xi$ parameter), and manage to reproduce the observed quasi-periodicity and off-states of Vela X-1. Meanwhile, in all of our simulations, quasi-periodic behavior or off-state is not observed, and it is unlikely that this is due to the difference between 2D and 3D. This suggests that radiation of the NS is crucial to quasi-periodicity and off-states.

7.4.3 Dynamics near and inside the magnetosphere

All our simulations ignores the effect of NS magnetic field, whose effect can be important for $r \lesssim R_{\text{mag}}$. Therefore, our simulations are only meaningful for flow outside the magnetosphere (and we do achieve $r_{\text{in}} \sim R_{\text{mag}}$ in some cases). What happens inside the magnetosphere can directly affect the accretion rate and its variability, and feedback (e.g. outflows and jets) from inside the magnetosphere may modify the dynamic of larger scale flows.

Accretion near and inside the magnetosphere has been discussed, for example, in Shakura et al. (2013), which considers quasi-spherical accretion (i.e. the outer boundary condition is a radial, laminar inflow). An extended quasi-static shell is formed on the magnetosphere, and the accretion flow penetrates the magnetosphere through Rayleigh-Taylor instabilities. However, in reality the flow outside the magnetosphere can often be turbulent. In that case, strong turbulence may make penetrating the magnetosphere easier, and the overdense shell may not need to form. On the other hand, if an overdense shell still forms on the magnetosphere, it could serve as a “buffer” and reduce the turbulent variation of the accretion rate.

Moreover, as the magnetosphere tends to obstruct inflow, it may be equivalent to a partially absorbing (and partially reflecting) accretor. This obviously can reduce the accretion rate. Additionally, it may impact to the stability of the flow. In axisymmetric simulations it is known that a non-absorbing accretor (i.e. a solid gravitating sphere) makes the flow unstable by creating vortices (Matsuda et al., 1989); it is likely that such effects happens in 3D for a partially absorbing accretor as well, and the production of outflow / vortices can eventually lead to turbulence.

8 Conclusion

In this paper, we investigate BHL accretion with and without transverse upstream gradient using 2D axisymmetric and 3D hydrodynamic simulations. We use a $\gamma = 5/3$ adiabatic equation of state, and focus on the regime of high (upstream) Mach number, weak upstream gradient and small accretor size; this regime is relevant to most observed wind-fed SgXBs but has not been systematically explored before.
When there is no upstream gradient, 2D axisymmetric (§4) and 3D (§5.2) simulations at different Mach number and accretor size ($r_{in}$) all show stable, axisymmetric accretion flow. The “flip-flop” instability commonly observed in 2D planar BHL accretion does not occur in 3D.

When the upstream gradient is small but nonzero (§5.3), however, the flow is significantly more prone to instability than previously expected. When there is only upstream density gradient ($\epsilon_\rho$), the flow becomes unstable at $\epsilon_\rho^2 > r_{in}/50R_a$ (§5.4). When unstable, the flow near the accretor eventually becomes highly turbulent. The accretion rate ($\dot{M}$) and mean (averaged over the accretor but not over time) specific angular momentum of accreted material ($\langle L_z \rangle$) depend on $r_{in}$ but barely on $\epsilon_\rho$. As $r_{in}$ decreases, the time-averaged $\dot{M}$ and $\langle L_z \rangle$ decrease non-convergently and their variation increase.

As long as the upstream gradient is small, no persistent disk can form (§5.5). The turbulent flow near the accretor screens the effect of upstream gradient and suppresses disk formation, and a persistent disk-like structure should form only when $\epsilon_\rho^2 > r_{in}/R_a$, with $R_{\text{shock}}$ being the shock standoff distance (§7.2, 7.3). This requires significantly larger upstream gradient compared to simple analytic estimates, which often suggest that disk should form when $\epsilon_\rho^2 > r_{in}/R_a$.

To apply our results to real SgXBs, we develop a model which adopts a more realistic wind profile and includes orbital effects (§2.2, §6.1). We perform two sets of simulations with this model, one resembling Vela X-1 with (likely unrealistic) fast wind, the other resembling OAO 1657-415. These two systems serve as examples of small and large upstream gradient respectively. For Vela X-1 with fast wind (§6.2), we observe a stable accretion flow down to $r_{in} = 0.0025R_a$; this is more stable compared to simulations with only upstream density gradient. This trend of increased stability when adopting a realistic wind model and including orbital effects is also observed in previous studies, but its reason remains unclear. (§7.3). For OAO 1657-415 (§6.3), the large upstream gradient leads to the formation of a persistent but turbulent and geometrically thick disk-like structure, agreeing with our analytic estimate.

Regimes of different behaviors in ($\epsilon_\rho, r_{in}/R_a$) parameter space are summarized in §7.3. Overall, the accretion flow is more prone to instability and disk is less likely to form than previously expected. The regime of turbulent and disk-less flow occupies a large parameter space, and most observed SgXBs lie inside this regime. Therefore, studying small-scale ($r \lesssim R_{\text{mag}}$) NS accretion from a turbulent flow (rather than a thin accretion disk or quasispherical laminar inflow) is necessary.

References


\[17\] Meanwhile, simple analytic estimates often suggest that the flow can be unstable without disk-like structure only when $\epsilon_\rho^2 \sim r_{in}/R_a$ (which is a very small region in the parameter space) since (naively) smaller $\epsilon_\rho$ should give stable flow and larger $\epsilon_\rho$ should allow disk formation.