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WHAT IS NATURAL FERTILITY? THE REMODELING OF A CONCEPT

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The concept of natural fertility has proved useful in demographic research, although the dispute over its applicability to real populations still persists (for reviews, see Knodel, 1983; Leridon, 1988; Wilson et al., 1988). One of the most important extensions of Henry's original conceptualization of natural fertility was made by Coale (1971) and Coale and Trussell (1974, 1975) in the framework of model fertility schedules. Coale and Trussell characterized age-specific fertility rates of any population by two parameters as deviations from a set of standard natural fertility rates. One parameter, M, measures the departure of the population concerned in the level of fertility; another parameter, m, measures the degree of fertility control, which has a built-in increasing function with age. Wilson et al. (1988) use Coale and Trussell's method to model natural fertility because natural fertility can always be seen as a special case of regular fertility.

Successful as it appears, this last project completes a questionable cycle. The Coale-Trussell model presumes perfect knowledge of a set of standard fertility rates. The use of the Coale-Trussell formulation to model natural fertility challenges this assumption. Furthermore, Wilson et al. (1988) apply the Coale-Trussell model to the same data from which Coale and Trussell (1974) obtained the standard natural fertility schedule; from the viewpoint of statistical estimation, there is less information in the data than imposed by this procedure. Finally, Wilson et al. (1988) forcefully and convincingly demonstrate the importance of treating the data as samples rather than as populations. If the data are treated as samples, as they should be, the natural fertility standard should also be statistically estimated, as are m and M parameters.

Henry (1961) as well as Coale and Trussell (1974) identified natural fertility as a peculiar *age pattern*. For physiological and social factors, different populations not exercising fertility control may have different levels of fertility. But their age patterns should be the same. This idea was evident in Henry's work and was formalized into the Coale-Trussell method. For the data initially reported by Henry (1961) and later used by Coale and Trussell (1974), differing levels of fertility with a common age pattern were assumed but, unfortunately, untested. Coale and Trussell (1974) obtained their natural fertility standard by averaging the ten natural fertility schedules reported by Henry (1961) that are believed to be reliable. In its later use, this standard is always treated as exactly known (e.g., Broström, 1985; Wilson et al., 1988).

This paper applies log-linear models to Henry's data on natural fertility. It tests various assumptions leading to ways of obtaining a standard natural fertility schedule through explicit modeling. The models specify that births follow an independent Poisson distribution for each age interval of each population. All parameters are estimated through an iterative maximum-likelihood procedure.

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Three Models of Natural Fertility

We reconsider the data used by Henry (1961) and Coale and Trussell (1974) in the form of rates. We use the data in the form of raw frequencies as they have recently been reconstructed by Wilson et al. (1988). Let us denote B for the number of births and T for the total exposure to birth in terms of women-years. The two sets of frequencies for B and T are contained in two 6 x 10 two-way tables indexed by a (a = 1,...,6) for A (age) and i (i = 1,...,10) for P (population). Note that six age intervals (20-24, 25-29, 30-34, 35-39, 40-44, and 45-49) are used.

We specify the saturated multiplicative model as follows:

$$(1)\quad B_{ai}^{AP} = T_{ai}^{AP} \cdot \tau_a^{A} \cdot \tau_i^{P} \cdot \tau_{ai}^{AP} ,$$

where τ^{A} is the marginal age effect, τ^{P} is the marginal population effect, and τ^{AP} is the interaction effect. The exposure T^{AP} is included in the model as a control. The basic idea is that the number of births equals the exposure times the fertility rate. All terms after T^{AP} in Equation (1) comprise the model for the fertility rate. Taking the natural logarithm on both sides of the equation, we have

$$(2)\quad \log(B_{ai}^{AP}) = \log(T_{ai}^{AP}) + \mu_a^{A} + \mu_i^{P} + \mu_{ai}^{AP} ,$$

where μ's are log transformations of τ parameters. Equation (2) is called the "loglinear" model. For identification purposes, normalizations on μ (or τ) parameters are necessary. Of sixteen marginal parameters, six μ^{A}'s (or τ^{A}'s) and ten μ^{P}'s (τ^{P}'s), only fifteen are free. We add one normalization constraint:

$$(3)\quad \sum_{i=1}^{10} \mu_i^{P} = 0 .$$

As will become clear, Equation (3) is equivalent to the statement that the geometric mean of M_i is 1. This corresponds to Coale and Trussell's (1974) normalization rule that M is 1 for natural fertility. In the saturated model of (2), 45 out of a total of 60 interaction parameters (μ^{AP}'s) are identifiable. We consider restricted models requiring fewer than 45 parameters representing interactions.

Three models are described as follows. They are: (1) Homogeneity Model, (2) Independence Model, and (3) Fertility Control Model. These models are special cases of the general model represented by Equation (2).

<u>Homogeneity Model</u>

The Homogeneity Model can be stated in terms of μ parameters:

$$(4)\quad \mu_i^P = \mu_{ai}^{AP} = 0\ ,$$

for all possible a and i. The model assumes that there is virtually no difference, either in the level or in the pattern, among the ten different natural fertility populations. The observed differences are attributable to sampling error. This is a simplistic model, a model that was rejected by Henry (1961).

Independence Model

The Independence Model specifies the independence between age effects and population effects. There exists an overall population-specific factor that does not depend on age. That is,

$$(5)\quad \log(B_{ai}^{AP}) = \log(T_{ai}^{AP}) + \mu_a^A + \mu_i^P\ .$$

We can compute predicted fertility rates by:

$$(6)\quad n_{ai} = B_{ai}/T_{ai} = \exp(\mu_a^A + \mu_i^P) = \exp(\mu_a^A)\cdot\exp(\mu_i^P) = n_a\cdot M_i\ ,$$

where n_a ($n_a = \exp(\mu_a^A)$) is defined as the common age pattern of natural fertility and M_i ($M_i = \exp(\mu_i^P)$) is the measure of fertility level. Implicit in the Independence Model is a proportionality constraint. The ratio in age-specific fertility rates between a natural fertility population and the natural fertility standard or between any two natural fertility populations is a constant across age: $n_{ai}/n_a = M_i$ and $n_{ai}/n_{aj} = M_i/M_j$, for $a = 1,\ldots,6$.

Fertility Control Model

The Fertility Control Model treats the ten natural fertility populations as regular populations. The model utilizes the general form of the Coale-Trussell method and therefore allows for fertility control:

$$(7)\quad \log(B_{ai}^{AP}) = \log(T_{ai}^{AP}) + \mu_a^A + \mu_i^P + m_i\cdot v_a$$

Equation (7) satisfies the Coale-Trussell specification because

$$(8)\quad n_{ai} = B_{ai}/T_{ai} = \exp(\mu_a^A + \mu_i^P + m_i\cdot v_a) = \exp(\mu_a^A)\cdot\exp(\mu_i^P)\cdot\exp(m_i\cdot v_a)$$
$$= n_a\cdot M_i\cdot\exp(m_i\cdot v_a)\ .$$

For the current analysis, we assume that v_a is a set of known parameters.[1] One normalization constraint on m_i's is needed because only nine out of ten m_i's are free. Following Coale and Trussell's (1974) practice of setting m to zero for natural fertility, we constrain m_i's so that $\Sigma m_i = 0$.

Results

The three models as expressed by Equations (4), (5), and (7) can be estimated by assuming that births for each age interval in each population follow a Poisson distribution. The models are estimated according to the maximum-likelihood principle. The goodness of fit of the models is first measured by the log-likelihood ratio test statistic, L^2. Asymptotically, L^2 and the difference in L^2 between two nested models follow the chi-square distribution. However, it is well-known that, with large samples, the log-likelihood ratio test is likely to reject a good model. Raftery (1986) proposes a Bayesian statistic, *BIC*, for large samples: $BIC = L^2 - (DF)\log N$, where L^2 is the log-likelihood ratio statistic, *DF* is the associated degrees of freedom, and *N* is the sample size. If *BIC* is negative, we should accept the null hypothesis that the model is true. When comparing several models, we should select the model with the lowest *BIC* value. For a descriptive measurement of the goodness of fit, we also use the Index of Dissimilarity, denoted as *D*. The Index of Dissimilarity here can be interpreted as the proportion of misclassified births.

The goodness of fit statistics for the three estimated models are reported in Table 1. It is evident that the Homogeneity Model does not fit the data ($L^2 = 658.92$ for 54 degrees of freedom, $BIC = 62.18$). The Independence Model fits the data much better. The L^2 statistic is reduced by 430.70 for 9 degrees of freedom, meaning that the population effects are highly significant. Judged by the L^2 statistic alone, the Independence Model does not fit the data very well (228.22 with 45 degrees of freedom). This can be accounted for by the large sample size (62,987). Adjusting for sample size, the Independence Model fits the data quite well. This can be shown first by a negative value of the *BIC* statistic (-269.07) for the model. Additional evidence is given by the Index of Dissimilarity. The Independence Model misclassifies only 1.14 percent of the total births, a substantial reduction from 2.45 percent for the Homogeneity Model. The improvement of the Fertility Control Model is significant according to the difference in the L^2 statistic (80.95 for 9 degrees of freedom). However, according to the *BIC* statistic, the Fertility Control Model is not as good as the Independence Model (-250.56 versus -269.07). The reduction in misclassifications is also marginal (from 1.14 to 1.04

Table 1: Loglinear Models for Henry's (1961) Natural Fertility Data

Model	Description	L^2	*DF*	*D*	*BIC*
Homogeneity	*A*	658.92	54	2.45%	62.18
Independence	*A* + *P*	228.22	45	1.14%	-269.07
Fertility Control	*A* + *P* + *PV*	147.27	36	1.04%	-250.56

Note: *A* = age; *P* = population; *V* = vector of fertility control parameters (Coale and Trussell, 1974). L^2 is the log-likelihood ratio chi-square statistic with the degrees of freedom reported in column *DF*. *D* is the index of dissimilarity. $BIC = L^2 - (DF)\log(N)$, where *N* is the total number of births (62,987).

percent). Overall, it is safe to conclude that the Independence Model fits the data well. The improvement in the goodness of fit for allowing fertility control is discernible but marginal. As shown in Figure 1 for a typical case (Hutterites, 1921-1930), the Independence Model performs just as well as the Fertility Control Model in predicting the observed fertility rates. In contrast, the Homogeneity Model performs poorly.

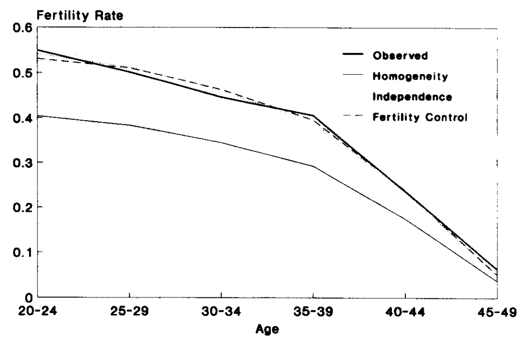

Figure 1: Observed and Predicted Rates, Hutterites, 1921-1930

Table 2 reports standard age-specific natural fertility rates estimated by different methods. In Panels A and B, fertility rates are obtained from the three loglinear models. Rates in Panel A are unstandardized in the sense that the normalization condition of Equation (3) is used. The unstandardized rates reported in Panel A are difficult to compare across the three models. Since we are interested only in age patterns and not in levels of fertility, we standardize the three sets of rates so that fertility rate is 0.460 for the first age interval. This standardization essentially changes the normalization condition of Equation (3). The rates thus standardized are displayed in Panel B. For

Table 2: Age-Specific Fertility Rates under Different Models

Model	Age 20-24	25-29	30-34	35-39	40-44	45-49
			Panel A: Unstandardized			
Homogeneity	0.404	0.384	0.345	0.293	0.175	0.037
Independence	0.441	0.417	0.376	0.319	0.191	0.041
Fertility Control	0.472	0.428	0.357	0.282	0.157	0.032
			Panel B: Standardized			
Homogeneity	0.460	0.437	0.393	0.333	0.198	0.042
Independence	0.460	0.436	0.392	0.333	0.199	0.043
Fertility Control	0.460	0.417	0.349	0.275	0.153	0.031
			Panel C: Coale-Trussell			
Before Henry's Adjustment	0.459	0.431	0.394	0.322	0.169	0.024
After Henry's Adjustment	0.460	0.431	0.395	0.322	0.167	0.024

Note: Rates in Panels A and B are based on estimates of loglinear models reported in Table 1. Rates in Panel C are simple averages across the ten populations used by Coale and Trussell (1974). Henry (1961) adjusted the observed rates for two populations (Sotteville-les-Rouen and Crulai) for unknown reasons (see Wilson et al., 1988:10).

comparison, rates obtained by the averaging method are presented in Panel C. There are two sets of rates in Panel C. One is based on the original data. Another is based on the data as adjusted by Henry (1961) for two populations (Sotteville-les-Rouen and Crulai) (see Wilson et al., 1988:10).

There is virtually no difference between the standardized rates from the Homogeneity Model and those from the Independence Model. The standardized rates from the Fertility Control Model have a steeper decreasing shape than do those from the other two models. The effect of Henry's adjustment to the data is small, as the two sets of rates in Panel C are similar. Let us compare the second row of Panel B and the second row of Panel C. The former is the new standard pattern of natural fertility; the latter is the old standard that has been in use since Coale and Trussell (1974). We recommend the replacement of Coale and Trussell's standard with the new standard estimated from the Independence Model. The main difference between the two is that in later age intervals (35-39, 40-44, and 45-49), fertility rates of the new standard are higher than those of the old standard.

The estimates of M and m by different methods are presented in Table 3. We compare four methods: (1) the Independence Model, (2) the Fertility Control Model, (3) Broström (1985) and Wilson et al. (1988), and (4) Coale and Trussell (1978). X^2 is the Pearson chi-square statistic, a

Table 3: Estimates of M and m for Henry's (1961) Natural Fertility Data

Population	Method (1)	Method (2)	Method (3)	Method (4)
Estimates of M				
Hutterite 1921-1930	1.229	1.126	1.131	1.148
Canada 1700-1730	1.219	1.112	1.109	1.103
Hutterite before 1921	1.100	1.022	1.018	1.016
Bourgeoisie of Geneva 1600-1649	1.059	1.180	1.186	1.186
Europeans of Tunis 1840-1859	1.036	1.000	1.006	0.989
Sotteville-les-Rouen (Normandy) 1760-1790	0.989	1.182	1.161	1.109
Crulai (Normandy) 1674-1742	0.930	0.981	0.978	0.986
Norway 1874-1876	0.914	0.831	0.813	0.835
Bourgeoisie of Geneva before 1600	0.843	0.852	0.853	0.862
Taiwan (about 1900)	0.780	0.806	0.806	0.805
Estimates of m				
Hutterite 1921-1930		-0.147	-0.114	-0.082
Canada 1700-1730		-0.152	-0.140	-0.152
Hutterite before 1921		-0.123	-0.117	-0.125
Bourgeoisie of Geneva 1600-1649		0.209	0.231	0.236
Europeans of Tunis 1840-1859		-0.047	-0.021	-0.065
Sotteville-les-Rouen (Normandy) 1760-1790		0.234	0.236	0.207
Crulai (Normandy) 1674-1742		0.085	0.100	0.108
Norway 1874-1876		-0.153	-0.170	-0.130
Bourgeoisie of Geneva before 1600		0.028	0.045	0.074
Taiwan (about 1900)		0.066	0.075	0.071
X^2	187.68	126.43	513.90	546.68
Degrees of Freedom	45	36	36	36

Note: Method 1 = Independence Model; Method 2 = Fertility Control Model; Method 3 = Broström (1985) and Wilson et al. (1988); Method 4 = Coale and Trussell (1978). X^2 is the Pearson chi-square statistic, which equals to the sum of all $(O_{ai} - E_{ai})^2/E_{ai}$, where O_{ai} is the observed births, and E_{ai} is the expected births.

sensible measure for comparing the relative performances of different methods.[2] As shown in Table 3, the Independence Model and the Fertility Control Model yield estimates of m and M that predict observed births far better than the other two methods. Note that the total degrees of freedom reported by the computer program for Methods 3 and 4 are incorrect (40 instead of 36) because the standard natural fertility schedule is estimated from the very same data and cannot be taken as exactly known.[3]

Discussion and Conclusion

We conclude that the Independence Model fits the data well and should be accepted as the final model. The Homogeneity Model is unambiguously rejected; the Fertility Control Model is unsupported by the data. The Independence Model states, as shown in Equation (6), that natural fertility populations exhibit a common *age pattern* but may have different *levels* of fertility. This was exactly the same conclusion of Henry's (1961) original work and was subsequently embedded into Coale and Trussell's (1974) model fertility schedules. Thus this paper echoes Leridon's (1988) defense for the original definition of natural fertility. It is unfruitful to estimate the fertility control parameter (m) for natural fertility populations, as it has been done in Coale and Trussell (1978) and Wilson et al. (1988).

It is a well-known statistical principle that the more observations there are in a sample, the more reliable a sample statistic is, everything else being equal. Here reliability is defined as the variance of the sample statistic in its sampling distribution. The method of averaging (Coale and Trussell, 1974) assumes that the observed rates are equally reliable across the ten populations. From Wilson et al. (1988), we have learned that this is not true. The sample sizes of the natural fertility data are quite different, varying from 185 (Sotteville-les-Rouen) to 51,146 (Norway). That is, the estimated age-specific rates (n_{ai}) for different populations have different degrees of reliability. The overall rates (n_a) should give more weights to large samples than to small samples because the former are less prone to sampling errors than the latter.

The new estimates are superior to the old ones for a number of reasons. First, the new estimates are estimated by an explicit model (the Independence Model), which is tested against two alternative models (the Homogeneity Model and the Fertility Control Model). Additionally, because of the empirical acceptance of the Independence Model, the fertility control parameter (m_i) is set to zero for a more efficient estimation of other parameters. Finally, the maximum-likelihood estimation procedure is such that larger samples weigh more than small samples in determining the standard natural fertility schedule (n_a). Therefore, the future user of the Coale-Trussell method is advised to take advantage of the new estimates of the standard natural fertility function.

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NOTES

[1] We use the values reported by Coale and Trussell (1974). Admittedly there are problems with Coale and Trussell's estimation method of obtaining these values, as noted by Xie (1990). Because these v values are widely accepted and do not appear to be much different from those reestimated by Xie (1990), Coale and Trussell's v values are therefore used.

[2] The log-likelihood ratio chi-square statistic does not apply here, first because Methods 2 and 3 are not nested, and second because Method 4 is not a maximum-likelihood estimation.

[3] For this reason, the confidence regions of M and m reported by Wilson et al. (1988) are underestimated.

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