Graduate Public Finance
Overview of Spatial Public Finance and the Rosen-Roback Model

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Princeton
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Lecture 3
What’s special about Spatial PF?

- Mobility of factors (and goods)
- Spillovers
  - Agglomeration
  - Congestion
- Spatial Heterogeneity in Endowments (and Outcomes)
- Hierarchy
  - Federalism
  - Competition with many neighbors
Academic Motivation:

1. Key policy debates, large spatial disparities, labs of democracy
2. Rich setting for economics and great data
3. Overlap w/ many fields (labor, urban, trade, development, macro)

Goals:

1. Provide context and guidance on open questions
2. Present benchmark models and new research
3. Enhance your applied modeling and empirical skills
Questions

1. Taxation: how should we pay for government services?
   - What should we tax? With what structure? At what rate?
   - Taxation of capital, labor, and goods in a spatial setting
   - Incidence, efficiency, and policy implications

2. Spending: how big should government be and what should it provide?
   - Are local services being under or over provided (level and composition)?
   - How are local services allocated? E.g., How much police spending allocated to rich/poor neighborhoods?
   - Redistribution, safety net, and mobility responses to benefit generosity

3. Hierarchy: How should governments be organized?
   - When is local provision efficient?
   - Fiscal federalism and Tax Competition

4. Dynamics: Growth, Economic Development, and Poverty
   - Big push and Industrial policy? Local vs Aggregate Consequences?
   - Should we have special economic zones? Bail outs? Pension reform?
   - Opportunity and growth across locations: causes, consequences, and policy implications
Motivation: Geographically concentrated economic activity

Figure 1  Spatial distribution of economic output in the US, by square mile. Notes: This figure reports the value of output produced in the US by square mile.

Source: Moretti (2011)
Motivation: Geographically concentrated upward mobility

A. Absolute Upward Mobility: Average Child Rank for Below-Median Parents ($\bar{y}_{25}$) by CZ

Source: Chetty-Hendren-Kline-Saez (2014)
Motivation: Geographically concentrated poverty
Motivation: Geographically concentrated poverty/race

Motivation: Geographically concentrated poverty/race

The same data, aggregated by community area and shown with solid colors.

Motivation: Geographically concentrated shocks

The Parts of America Most Vulnerable to China

Some areas of the U.S. were hit especially hard by China’s rise, partly because those areas had lots of jobs in industries where imports surged the most.

Most-affected areas of the U.S.

Colors show which areas were most affected by China’s rise, based on the increase in Chinese imports per worker in each area from 1990 to 2007. Hovering over each area on the map will show a demographic breakdown of that area, below, and its most-affected industries, at right.

Most-affected industries

<table>
<thead>
<tr>
<th>Most-affected industries</th>
<th>Impact per worker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Furniture and fixtures</td>
<td>$44k</td>
</tr>
<tr>
<td>Games, toys, and children’s vehicles</td>
<td>$488k</td>
</tr>
<tr>
<td>Sporting and athletic goods</td>
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</tr>
<tr>
<td>Electronic components</td>
<td>$65k</td>
</tr>
<tr>
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<td>$11k</td>
</tr>
<tr>
<td>Motor-vehicle parts and accessories</td>
<td>$12k</td>
</tr>
<tr>
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<td>$202k</td>
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</table>
Motivation: Geographically concentrated shocks

Furniture and fixtures

Most-affected areas of the U.S.
Colors show which areas were most affected by China’s rise, based on the increase in Chinese imports per worker in each area from 1990 to 2007. Hovering over each area on the map will show a demographic breakdown of that area, below, and its most-affected industries, at right.

Most-affected industries

<table>
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<th>Industry</th>
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<th>Impact per worker†</th>
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<tr>
<td>Furniture and fixtures</td>
<td>196 areas</td>
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<tr>
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<td>106 areas</td>
<td>$82k</td>
</tr>
<tr>
<td>Electronic components</td>
<td>87 areas</td>
<td>$65k</td>
</tr>
<tr>
<td>Plastics products</td>
<td>84 areas</td>
<td>$11k</td>
</tr>
<tr>
<td>Motor-vehicle parts and accessories</td>
<td>79 areas</td>
<td>$12k</td>
</tr>
<tr>
<td>Electronic computers</td>
<td>68 areas</td>
<td>$20k</td>
</tr>
</tbody>
</table>

Source: Autor Dorn Hanson http://chinashock.info
Motivation: Geographically concentrated shocks
Motor-vehicle parts and accessories

Most-affected areas of the U.S.
Colors show which areas were most affected by China’s rise, based on the increase in Chinese imports per worker in each area from 1990 to 2007. Hovering over each area on the map will show a demographic breakdown of that area, below, and its most-affected industries, at right.

Most-affected industries
Most-affected industries, based on number of areas*  Impact per worker†

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Source: Autor Dorn Hanson http://chinashock.info
Motivation: Geographically concentrated shocks

Demographics of the most-affected areas

They were whiter, less educated, older and poorer than most of the rest of America. The bars below show those demographics by percentage of the population.

<table>
<thead>
<tr>
<th>Demographics of the most-affected areas</th>
<th>Non-Hispanic whites</th>
<th>High-school education or less</th>
<th>At least 50 years old</th>
<th>Below 150% of the poverty line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most-affected 20%</td>
<td>72.7%</td>
<td>45.6%</td>
<td>35.5%</td>
<td>27.6%</td>
</tr>
<tr>
<td>Least-affected 20%</td>
<td>67.3%</td>
<td>45.3%</td>
<td>34.3%</td>
<td>27.1%</td>
</tr>
<tr>
<td>Second-lowest 20%</td>
<td>64.7%</td>
<td>41.2%</td>
<td>34.0%</td>
<td>25.4%</td>
</tr>
<tr>
<td>Second-highest 20%</td>
<td>62.2%</td>
<td>40.9%</td>
<td>32.9%</td>
<td>24.8%</td>
</tr>
<tr>
<td>Middle 20%</td>
<td>58.4%</td>
<td>40.5%</td>
<td>32.5%</td>
<td>24.5%</td>
</tr>
</tbody>
</table>

*Number of areas is based on the number of commuting zones where each industry was among the five most-affected industries. Commuting zones are groups of counties that share a labor market, similar to a metropolitan area.
Impact per worker means the value of goods that a U.S. worker would have produced if those goods had been made in America instead of China.
The analysis excludes Alaska and Hawaii. Education level is measured only for those age 25 and over. Poverty-status calculations are for individuals, and exclude those in college dormitories and military housing, as well as institutionalized people and children under age fifteen who aren’t related to a householder or other reference person.

Graphic by Andrew Van Damm and Jessia Ma. Additional reporting by Jon Hilsenrath and Bob Davis.
Sources: David Autor and Brendan Price of the Massachusetts Institute of Technology; Gordon Hanson of the University of California, San Diego; David Dorn of the University of Zurich

Source: Autor Dorn Hanson Price http://chinashock.info
### Motivation: Geographically concentrated unemployment

Source: Kline Moretti (2013)

#### Overview of Spatial Public Finance

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**Table 1: Metropolitan Areas with the Highest and Lowest Unemployment Rates in 2008**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Metropolitan Area</th>
<th>Unemployment Rate (1)</th>
<th>Adjusted Unemployment Rate (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Flint, MI</td>
<td>.1462</td>
<td>.1399</td>
</tr>
<tr>
<td>2</td>
<td>Yuba City, CA</td>
<td>.1099</td>
<td>.1072</td>
</tr>
<tr>
<td>3</td>
<td>Anniston, AL</td>
<td>.1074</td>
<td>.0899</td>
</tr>
<tr>
<td>4</td>
<td>Merced, CA</td>
<td>.1060</td>
<td>.0948</td>
</tr>
<tr>
<td>5</td>
<td>Toledo, OH/MI</td>
<td>.1058</td>
<td>.1064</td>
</tr>
<tr>
<td>6</td>
<td>Yakima, WA</td>
<td>.1047</td>
<td>.0970</td>
</tr>
<tr>
<td>7</td>
<td>Detroit, MI</td>
<td>.1044</td>
<td>.1082</td>
</tr>
<tr>
<td>8</td>
<td>Chico, CA</td>
<td>.1031</td>
<td>.1092</td>
</tr>
<tr>
<td>9</td>
<td>Modesto, CA</td>
<td>.1027</td>
<td>.1021</td>
</tr>
<tr>
<td>10</td>
<td>Waterbury, CT</td>
<td>.1023</td>
<td>.0918</td>
</tr>
<tr>
<td></td>
<td><strong>Areas with the Lowest Rate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>276</td>
<td>Provo-Orem, UT</td>
<td>.0391</td>
<td>.0369</td>
</tr>
<tr>
<td>277</td>
<td>Madison, WI</td>
<td>.0389</td>
<td>.0511</td>
</tr>
<tr>
<td>278</td>
<td>Odessa, TX</td>
<td>.0383</td>
<td>.0307</td>
</tr>
<tr>
<td>279</td>
<td>Fargo-Morehead, ND/MN</td>
<td>.0362</td>
<td>.0467</td>
</tr>
<tr>
<td>280</td>
<td>Charlottesville, VA</td>
<td>.0348</td>
<td>.0362</td>
</tr>
<tr>
<td>281</td>
<td>Houma-Thibodoux, LA</td>
<td>.0337</td>
<td>.0107</td>
</tr>
<tr>
<td>282</td>
<td>Billings, MT</td>
<td>.0304</td>
<td>.0324</td>
</tr>
<tr>
<td>283</td>
<td>Rochester, MN</td>
<td>.0297</td>
<td>.0292</td>
</tr>
</tbody>
</table>
Motivation: Geographically concentrated unemployment
Differences are persistent ($\rho = .59$)

Figure 1: Unemployment Rates in 1990 and 2008, by Metropolitan Area

Notes: Data are from the 1990 Census of Population and the 2008 American Community Survey. The sample includes all individuals in the labor force between the age of 14 and 70.

Source: Kline Moretti (2013)
Motivation: Geographically concentrated unemployment
Convergence is slowing

Source: Ganong and Shoag (2014)
Motivation: Geographically concentrated recessions

Source: Yagan (2016)
Motivation: Geographically concentrated policy responses

Maximum Duration of Unemployment Insurance by State

- States with fewer than 26 weeks of regular benefits have proportionally fewer weeks of federal benefits available for those who file for UI after the reduction took effect. Please see the table on page 3 for a fuller explanation of the benefits available in each state.

Source: CBPP (2012)
Effects on political polarization (and many other outcomes)

Source: Autor Dorn Hanson Majlesi (2017) http://chinashock.info. "Congressional districts exposed to larger increases in import penetration disproportionately removed moderate representatives from office in the 2000s. Trade-exposed districts with an initial majority white population or initially in Republican hands became substantially more likely to elect a conservative Republican, while trade-exposed districts with an initial majority-minority population or initially in Democratic hands became more likely to elect a liberal Democrat"
Stakes are high...

For low-income people, life expectancy is highest in California, New York, and Vermont. It is lowest in Nevada. The next 8 states with the lowest life expectancies form a belt connecting Michigan, Ohio, Indiana, Kentucky, Tennessee, Arkansas, Oklahoma, and Kansas.

Source: https://healthinequality.org
Stakes are high...

Life expectancy varies substantially across cities, especially for low-income people. For the poorest Americans, life expectancies are 6 years higher in New York than in Detroit. For the richest Americans, the difference is less than 1 year.
Spatial Public Finance Outline

1. Baseline Rosen-Roback spatial model
2. Place-based Policies: theory
3. Place-based Policies: evidence
4. Sorting, fiscal federalism
Graduate Public Finance
The Rosen-Roback Spatial Model

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   - Overview
   - Workers: Indirect Utility Condition
   - Firms: No Profit Condition

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   - Components of Economic Models
   - Exogenous Model Parameters
   - Endogenous Model Outcomes
   - Equilibrium: Indifference Conditions
   - Solving Model

3 Comparative Statics and Value of Amenities
   - Price effects under different assumptions about amenities
   - Inferring Amenity Values
   - Extensions (Albouy JPE, 2009)
Model
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Overview

1. Goals
   - Characterize effect of amenity $s$ change on prices (wages and rents)
   - Infer the value of amenities

2. Markets
   - Labor: price $w$, quantity $N$
   - Land: price $r$, quantity $L = L^w + L^p$ for workers and production
   - Goods: price $p = 1$, quantity $X$

3. Agents
   - Workers (homogenous, perfectly mobile)
   - Firm (perfectly competitive, CRS)

4. Indifference Conditions
   - Workers have same indirect utility in all locations
   - Firm has zero profit (i.e., unit costs equal 1)
Utility is $u(x, l^c, s)$
- $x$ is consumption of private good
- $l^c$ is consumption of land
- $s$ is amenity

Budget constraint is $x + rl^c - w - \ell = 0$
- $\ell$ is non-labor income that is independent of location (e.g., share of national land portfolio)
- $w$ is labor income (note: no hours margin).
Workers: Indirect Utility

- Indirect utility is given

\[ V(w, r, s) = \max_{x, l^c, s} u(x, l^c, s) \text{ s.t. } x + r l^c - w - l = 0 \]

- Let \( \lambda = \lambda(w, r, s) \) be the marginal utility of a dollar of income, then

\begin{align*}
V_w &= \lambda > 0 \\
V_r &= -\lambda l^c < 0 \\
\Rightarrow V_r &= -V_w l^c
\end{align*}
Aside: Example of Indirect Utility

Utility is Cobb Douglas over goods and land with an amenity shifter:

\[ u(x, l^c, s) = s^{\theta w} x^\gamma (l^c)^{1-\gamma} \]

- Then \( x = \gamma \left( \frac{w+l}{1} \right) \) and \( l^c = (1 - \gamma) \left( \frac{w+l}{r} \right) \)
- So indirect utility is:

\[
V(w, r, s) = \gamma^\gamma (1 - \gamma)^{1-\gamma} \underbrace{s^{\theta w} 1-\gamma r^{-(1-\gamma)}}_{\text{constant}} \underbrace{1-\gamma r^{-(1-\gamma)}}_{\text{Amenities}} \underbrace{(w + l)}_{\text{Prices}} \underbrace{l^c}_{\text{Income}}
\]

- MU of income is \( \lambda(w, r, s) \)

\[
V_w = \lambda = \gamma^\gamma (1 - \gamma)^{1-\gamma} s^{\theta w} 1-\gamma r^{-(1-\gamma)}
\]

\[
V_r = -\lambda l^c = -\gamma^\gamma (1 - \gamma)^{1-\gamma} s^{\theta w} 1-\gamma r^{-(1-\gamma)} (1 - \gamma) \left( \frac{w + l}{r} \right)
\]

\[
\Rightarrow V_r = -V_w l^c
\]
Firms: Unit Cost Function

CRS production with cost function $C(X, w, r, s)$

- $X$ is output
- Unit cost $c(w, r, s) = \frac{C(X, w, r, s)}{X}$
- $L^p$ is total amount of land used by firms
- $N$ is total employment

From Sheppard’s Lemma, we have

$$c_w = \frac{N}{X} > 0$$
$$c_r = \frac{L^p}{X} > 0$$
Aside: Example technology, cost function, factor demand

Suppose \( X = f(N, L^p) = s^{\theta_F} N^\alpha L^{1-\alpha} \), then cost function is:

\[
C(X, w, r, s) = X (s^{\theta_F})^{-1} w^{\alpha} r^{1-\alpha} (\alpha^{-\alpha} (1 - \alpha)^{-1}) \
\Rightarrow 
\]

\[
c(w, r, s) = (s^{\theta_F})^{-1} w^{\alpha} r^{1-\alpha} (\alpha^{-\alpha} (1 - \alpha)^{-1}) \
\]

Then

\[
C_w(X, w, r, s) = \alpha \frac{(X (s^{\theta_F})^{-1} w^{\alpha} r^{1-\alpha} (\alpha^{-\alpha} (1 - \alpha)^{-1}))}{w} = N \\
C_r(X, w, r, s) = (1 - \alpha) \frac{(X (s^{\theta_F})^{-1} w^{\alpha} r^{1-\alpha} (\alpha^{-\alpha} (1 - \alpha)^{-1}))}{r} = L^p
\]

Dividing both sides by \( X \) gives:

\[
c_w = N/X > 0
\]
\[
c_r = L^p/X > 0
\]
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   - Extensions (Albouy JPE, 2009)
Aside: Components of Models

Three parts of any model

1. Exogenous parameters: model elements that are taken “as given”
2. Endogenous outcomes: model elements that “move around”
3. Equilibrium conditions: the set of rules that tells you what the endogenous model outcomes should be for a given set of exogenous model parameters.

“Given a [insert set of exogenous model parameters here], equilibrium is defined by the [insert endogenous model outcomes here] such that [list equilibrium conditions here].”
Exogenous parameters

- **Workers Parameters:** \( s, \theta_W, \gamma, I \)
  - \( s \) is level of amenities
  - \( \theta_W \) governs importance of amenities for utility
  - \( \gamma \) governs importance of goods for utility
  - \( 1 - \gamma \) governs importance of land for utility
  - \( I \) is non-labor income

- **Firm Parameters:** \( s, \theta_F, \alpha \)
  - \( s \) is level of amenities
  - \( \theta_F \) governs importance of amenities for productivity
  - \( \alpha \) is output elasticity of labor
  - \( 1 - \alpha \) is output elasticity of land
Recall:

- Labor: price $w$, quantity $N$
- Land: price $r$, quantities $L^w, L^p$ for workers and production
- Goods: price $p = 1$, quantity $X$

so endogenous outcomes are $w, r, N, L^w, L^p, X$
In equilibrium, workers and firms are indifferent across cities with different levels of $s$ and endogenously varying wages $w(s)$ and rents $r(s)$:

$$c(w(s), r(s), s) = 1$$  \hspace{1cm} (1)

$$V(w(s), r(s), s) = V^0$$  \hspace{1cm} (2)

where $V^0$ is the initial equilibrium level of indirect utility.

Specifically, in our example:

*Given $s, \theta_W, \theta_F, \gamma, I, \alpha$, equilibrium is defined by local prices and quantities $\{w, r, N, L^w, L^p, X\}$ such that 1 and 2 hold and land markets clear.*

N.B. We will mainly be focusing on prices: $w(s)$ and $r(s)$. 
Solving for effect of amenity changes on prices

- Differentiate 1 and 2 with respect to $s$ and rearrange, we have:

$$\begin{bmatrix} c_w & c_r \\ V_w & V_r \end{bmatrix} \begin{bmatrix} w'(s) \\ r'(s) \end{bmatrix} = \begin{bmatrix} -c_s \\ -V_s \end{bmatrix}$$

(3)

- Solving for $w'(s), r'(s)$, we have

$$w'(s) = \frac{V_r c_s - c_r V_s}{c_r V_w - c_w V_r}$$

$$r'(s) = \frac{V_s c_w - c_s V_w}{c_r V_w - c_w V_r}$$

- Note we can rewrite

$$c_r V_w - c_w V_r = \lambda L^p / X + \lambda I^c N / X = \lambda L / X = V_w L / X$$
Aside: example values for matrix elements

\[ c_w = \alpha \frac{(s^{\theta_F})^{-1} w^\alpha r^{1-\alpha} \kappa_0}{w} \]
\[ c_r = (1 - \alpha) \frac{(s^{\theta_F})^{-1} w^\alpha r^{1-\alpha} \kappa_0}{r} \]
\[ c_s = \theta_F \frac{(s^{\theta_F})^{-1} w^\alpha r^{1-\alpha} \kappa_0}{s} \]
\[ V_w = s^{\theta_W} 1^{-\gamma} r^{-(1-\gamma)} \kappa_1 \]
\[ V_r = -s^{\theta_W} 1^{-\gamma} r^{-(1-\gamma)} \kappa_1 (1 - \gamma) \left( \frac{w + l}{r} \right) \]
\[ V_s = \theta_W \frac{(s^{\theta_W} 1^{-\gamma} r^{-(1-\gamma)} \kappa_1 (w + l))}{s} \]

where \( \kappa_0 = \alpha^{-\alpha}(1 - \alpha)^{-(1-\alpha)} \) and \( \kappa_1 = \gamma^\gamma(1 - \gamma)^{(1-\gamma)} \) are constants.
Effect of amenity changes on prices

- Price changes

\[
\begin{align*}
  w'(s) &= \frac{(V_r c_s - c_r V_s)X}{\lambda L} \quad (4) \\
  r'(s) &= \frac{(V_s c_w - c_s V_w)X}{\lambda L} \quad (5)
\end{align*}
\]

- Special cases of interest:

1. Amenity only valued by consumers: \( \theta_F = 0 \Rightarrow c_s = 0 \)

2. Amenity only has productivity effect: \( \theta_W = 0 \Rightarrow V_s = 0 \)

3. Firms use no land \( 1 - \gamma = 0 \) and amenity is non-productive \( \theta_F = 0 \): \( c(w(s)) = 1, \ c_r = c_s = 0 \)
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1. Amenity only valued by consumers: $\theta_F = 0 \Rightarrow c_s = 0$

- When $c_s = 0$, higher $s \Rightarrow$ higher $r$, lower $l$

- Workers are willing to pay more in land rents and receive less in pay to have access to higher levels of amenities

\[
V(w, r, s^0) = V^0, \quad V(w, r, s^1) = V^0, \quad c(w, r) = 1
\]
2. Amenity only has productivity effect: $\theta_w = 0 \Rightarrow V_s = 0$

- When $V_s = 0$, higher $s \Rightarrow$ higher $r$ and higher $l$
- Firms are willing to pay more in land rents and wages to access higher productivity due to amenities

![Diagram](image-url)
3. Firms use no land $\gamma = 1$, amenity not productive $\theta_F = 0$

- Only production input is labor and firms are indifferent across locations, so wages must be the same across cities: $c(w(s)) = 1$

- Since $c_r = c_s = 0$,

  $$w'(s) = 0$$

  $$r'(s) = \frac{V_s c_w}{-c_w V_r} = \frac{V_s}{l^c V_w}, \text{ since } V_r = -l^c V_w$$

- So the rise in total cost of land for a worker living in a city with higher $s$ is

  $$l^c r'(s) = \frac{V_s}{V_w}$$
3. Firms use no land $\gamma = 1$, amenity not productive $\theta_F = 0$

- $\frac{V_s}{V_w}$ = marginal WTP for a change in $s$ so the marginal value of a change in the amenity is “fully capitalized” in rents

\[
\frac{V_s}{V_w} = \theta W \frac{(w+l)}{s}
\]

is increasing in income, decreasing in level of amenities
How do we infer the value of amenities in the more general case?

- $\Omega(s) = V(w(s), r(s), s)$ represents total utility of living in city $s$

- If all cities have equal utility, then

\[
\Omega'(s) = V_w w'(s) + V_r r'(s) + V_s = 0 \text{ in equilibrium}
\]

\[
V_s = -V_w w'(s) - V_r r'(s)
\]

\[
V_s = -V_w w'(s) + l^c V_w r'(s)
\]

\[
\implies \frac{V_s}{V_w} = l^c r'(s) - w'(s)
\]

- So WTP for the amenity is extra land cost for consumers less lower wages in a higher-amenity city
Inferring the Value of Amenities

We can get more insight from looking at firms:

- Firms face \( c(w(s), r(s), s) = 1 \) across cities, so

\[
c_{w}w'(s) + c_{r}r'(s) + c_{s} = 0 \quad (7)
\]

- Consider 2 cases

1. \( c_{s} = 0 \) (no productivity effects of higher amenity levels)
2. \( c_{s} \neq 0 \)
Inferring the Value of Amenities, $c_s = 0$

- In the case when $c_s = 0$,

$$w'(s) = \frac{-c_r}{c_w} r'(s)$$

$$= \frac{-L^p}{N} r'(s)$$

(8)

- Combine 6 and 7 to get the WTP of the $N$ people in a given city:

$$N \frac{V_s}{V_w} = Nl^c r'(s) + L^p r'(s) = L r'(s)$$

(9)

Thus, in this case, aggregate WTP can be derived from looking at how the total value of all land changes as $s$ changes.
Inferring the Value of Amenities, \( c_s \neq 0 \)

- Define “social value” \( SV \) as the sum of aggregate worker WTP and cost-induced savings. Then the change in \( SV \) given changes \( s \) is

\[
dSV = N \frac{V_s}{V_w} - Xc_s
\]

\[
= N(l^c r'(s) - w'(s)) - X(-c_w w'(s) - c_r r'(s))
\]

\[
= Nl^c r'(s) - Nw'(s)) + X \frac{N}{X} w'(s) + X \frac{L^P}{X} r'(s)
\]

\[
\Rightarrow dSV = Lr'(s)
\]

- So the change in social value is the change in total value of land
Extension: Albouy (JPE, 2009)

- Introduces a non-traded good \( y \) sold at city-specific price \( p \)
- Worker’s Problem: indirect utility is given by
  \[
  V(w, r, s) = \max_{x, y} u(x, y, s) \text{ s.t. } x + py - w - l = 0
  \]  
  (11)

- Unit cost function for tradable good:
  \[
  c(w, r, s) = 1
  \]  
  (12)

- Unit cost function for non-tradable good:
  \[
  g(w, r, s) = p
  \]  
  (13)

- Albouy model has 3 endogenous variables, \( w, r \) and \( p \), but can follow Rosen-Roback analysis
Extension: Albouy (JPE, 2009)

- Studies the unequal geographic burden of federal taxation
- Progressive fed tax schedule ⇒ higher taxes in higher \( w \) places
- “Federal taxes act like an arbitrary head tax for living in a city with wage improving attributes, whatever those attributes may be”
- Simulation: a worker moving from a typical low-wage city to a high-wage city would experience a 27% increase in federal taxes, which is equivalent to a $269 billion transfer from workers in high-wage, high-productivity areas to low-wage, low-productivity cities.

N.B. Could use approach to study an amenity \( s \) (e.g., inefficiency in the local construction sector) that raises the cost of the local good and has no inherent value for consumers or productivity effects on the traded sector (i.e., \( \theta_F = \theta_W = 0 \)).
Fig. 1.—Effect of federal taxes on a high trade-productivity city. In a simplified model \((r^j = p^j, Q^j = A^j = 1\) for all \(j\)), replacing a lump-sum tax, \(T\), with a utility-equivalent federal income tax, \(\tau\), raises wages, \(w\), and lowers rents, \(r\), and employment in Chicago, labeled “C,” a city with high trade productivity \((A_C^c > 1\)\), changing the equilibrium from \(E_0^c\) to \(E^c\).
Initial Equilibrium
- Zero profit condition is higher for Chicago due to higher TFP there
- without taxes, wages $w^C_0$ are higher in Chicago to pay for higher rents (note amenities are set equal in this example)

With progressive income taxes
- Workers in costlier cities like Chicago now need to be paid more to be willing to live there
- Relative to initial equilibrium, fewer workers in Chicago which lowers the demand for land in both production and consumption $\Rightarrow$ rents fall by $dr^C$
- This also raises the labor-to-land ratio, causing wages to rise $dw^C$
- Firms are no better off since cost savings on land are passed off to workers in higher wages
Moving to Miami: the higher quality of life case

Fig. 2.—Effect of federal taxes on a high-quality-of-life city. In a simplified model ($r^j = p^j, A_X^j = A_Y^j = 1$ for all $j$), replacing a lump-sum tax, $T$, with a utility-equivalent federal income tax, $\tau$, lowers wages, $w$, and raises rents, $r$, and employment in Miami, labeled “M,” a city with high quality of life ($Q^M > 1$), changing the equilibrium from $E_0^M$ to $E^M$.

Source: Albouy (JPE, 2009)
Initial Equilibrium

- Like Chicago, Miami is relatively crowded and has high rents, but as compensation, workers get a nicer environment rather than higher wages.
- Labor demand is downward sloping (due to fixed land supply) and a larger supply of workers means a lower equilibrium wage.
- Both cities have same TFP so on same zero-profit condition.
- The mobility condition is lower and to the right in Miami because of higher quality of life.

With progressive income taxes

- A worker is now more willing to bid down wage to live in Miami since a $1 wage cut implies only a $(1 - \tau)$ reduction in consumption.
- Relative to initial equilibrium, more workers in Miami which raises the demand for land in both production and consumption \( \Rightarrow \) rents increase by \( dr^M \).
- This also lowers the labor-to-land ratio, causing wages to fall \( dw^M \).