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Outline

1 Brief Introduction
   - What is capital income?
   - Sources of wealth and capital income
   - How is capital income taxed in the US?

2 Capital Tax Incidence: simple models and asset prices
   - Supply and demand in the capital market
   - Preliminaries rental and asset markets for capital
   - Application: Taxes, Housing, and Asset Markets (Poterba, 1984)
   - Simple spatial model: One factor, two locations

3 Capital Tax Incidence: Harberger
   - Fullerton and Ta (2017)
     - Welfare loss from taxation
     - Effect of Tax on Corporate Output
     - Effect of Tax on Capital
     - Effect of Tax on Corporate Capital
   - Harberger Model (more general utility and technology)
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Motivation

Equity-efficiency tradeoffs of capital taxation seem especially stark

1. **Efficiency**
   - Capital taxes reduce scale of economic activity
   - Capital accumulation, which may be highly responsive to rates of return, is correlated with economic growth
   - Capital mobility: taxes can lead to misallocation

2. **Equity**
   - Distribution of capital income is much more unequal than labor income
   - Capital mobility: burden may be shifted to labor

3. **Evidence**
   - Empirical evidence/our understanding of capital taxation is less well developed than labor income taxation

4. **Policy Relevance**
   - Future of fiscal policy (taxing robots, driverless cars, corp tax reform)
   - Destination-based cash flow taxes, international reforms
What is capital income?

Individuals derive market income (before tax) from labor and capital: \( z = wl + rk \) where \( w \) is wage, \( l \) is labor supply, \( k \) is wealth, \( r \) is rate of return on wealth.

1. **Labor income inequality** is due to differences in working abilities (education, talent, physical ability, etc.), work effort (hours of work, effort on the job, etc.), and luck (labor effort might succeed or not).

2. **Capital income inequality** is due to differences in wealth \( k \) (due to past saving behavior and inheritances received), and in rates of return \( r \) (varies dramatically over time and across assets).
Level and distribution of capital income (1/2)

- Labor income $wl \approx 75\%$ of national income $z$
- Capital income risk $rk \approx 25\%$ of national income $z$ (has increased in recent decades)
- Wealth stock $k \approx 400\% - 500\%$ of national income $z$ (is increasing)
- Rate of return on capital $r \approx 5\%$
- $\alpha = \beta \cdot r$ where $\alpha = rk/z$ share of capital income and $\beta = k/z$ wealth to income ratio
- In GDP, gross capital share is higher (35\%) because it includes depreciation of capital ($\approx 10\%$ of GDP)
- National Income = GDP − depreciation of capital + net foreign income
Level and distribution of capital income (2/2)

The top 1% share in the US: wealth vs. labor income

Source: Piketty, Saez and Zucman (2016).
Types of wealth and capital income

**Definition:** Capital Income = Returns from Wealth Holdings

- **Aggregate US Private Wealth** ≃ 4*Annual National Income
- **Housing:** residential real estate (land+buildings) \([\text{income} = \text{rents}]\) net of mortgage debt
- **Unincorporated business assets:** value of sole proprietorships and partnerships \([\text{income} = \text{individual business profits}]\)
- **Corporate equities:** Value of corporate stock \([\text{income} = \text{dividends} + \text{retained earnings}]\)
- **Fixed claim assets:** Currency, deposits, bonds \([\text{income} = \text{interest income}]\) minus debts \([\text{credit card, student loans}]\)
- **Pension funds:** Substantial amount of equities and fixed claim assets held indirectly through pension funds
Aggregate Household Wealth

Source: Saez Zucman (2019)
Components of Aggregate Household Wealth

A. Components of Aggregate Household Wealth

Source: Smith Zidar Zwick (2020)
Components of Aggregate Fiscal Capital Income

Source: Smith Zidar Zwick (2020)
Piketty Saez Zucman (2018)’s Composition of capital income

**FIGURE I**

From Taxable Income to National Income (1916–2014)

Source: Piketty Saez Zucman (2018)
Sources of wealth and capital income
Sources of wealth and capital income

Wealth = \( W \), Return = \( r \), Capital Income = \( rW \)

\[
W_t = W_{t-1} + r_t W_{t-1} + E_t + I_t - C_t
\]

where \( W_t \) is wealth at age \( t \), \( C_t \) is consumption, \( E_t \) labor income earnings (net of taxes), \( r_t \) is the average (net) rate of return on investments and \( I_t \) net inheritances (gifts received and bequests - gifts given).

Differences in Wealth and Capital income due to:

1. Age
2. past earnings, and past saving behavior \( E_t - C_t \) [life cycle wealth]
3. Net Inheritances received \( I_t \) [transfer wealth]
4. Rates of return \( r_t \)
Wealth over the lifecycle

Life Cycle Model

- Earnings
- Wealth
- Consumption
- Savings
- Dissaving

0: work starts
R: retirement
T: death

Time
Life cycle wealth versus Inherited wealth

1. **Life-cycle wealth** is wealth from savings earlier in your life
   - (e.g., pension contributions out of earnings, paying down a home mortgage, etc.)

2. **Inherited wealth** is wealth from inheritances received
   - (e.g., receiving a house or a trust fund from parents)

- Distinction matters for taxation because individuals are responsible for life-cycle wealth but not inherited wealth [meritocracy vs. aristocracy]
- Inherited wealth used to be very large in Europe (before World-War I), became small in post-World War II period, but is growing in recent decades (especially in Europe) Piketty (2014)

Analyzes income, wealth, inheritance data over the long-run:

- Growth rate $g = \text{population growth} + \text{growth per capita}$. Population growth will converge to zero, growth per capita for frontier economies is modest (1-1.5%) $\Rightarrow$ long-run $g \approx 1 - 1.5$

- Long-run aggregate wealth to income ratio ($\beta$) = savings rate ($s$) / annual growth ($g$):
  
  Proof: $W_{t+1} = (1 + g) \cdot W_t = W_t + s \cdot Y_t \Rightarrow W_t / Y_t = s / g$
  
  With $s = 8\%$ and $g = 2\%$, $\beta = 400\%$ but with $s = 8\%$ and $g = 1\%$, $\beta = 800\%$ $\Rightarrow$ Wealth will become important
Rate of return on wealth $r \simeq 5\%$ significantly larger than $g$ [except exceptional period of 1940s-1960s]

With $r \gg g$, role of inheritance in wealth grows and wealth inequality increases [past swallows the future]

- Explanation: Rentier who saves all her return on wealth accumulates wealth at rate $r$ bigger than $g$ and hence her wealth grows relative to the size of the economy. The bigger $r - g$, the easier it is for wealth to “snowball”: fortunes are created faster and last longer

$\Rightarrow$ Capital income taxation reduces $r$ to $r \cdot (1 - \tau_K)$ $\Rightarrow$ reduces wealth concentration and relative weight of inherited wealth
How much is it taxed?

In the US, total capital taxes can be decomposed into three categories of roughly equal importance from a macro perspective:

1. Corporate tax = 3% of \( Y \) (around 20% of a 15% tax base)
2. Annual property rates = 3% of \( Y \) (around 1% of a 300% tax base)
3. Personal taxes on a capital income = 2.8% of \( Y \) (around 30% of a 15% \times 60\% = 9\% tax base) + estates = 0.2% of \( Y \) (around 2% of a 10% tax base)

Won’t be able cover all of these in as much depth as I’d like, especially on optimal capital taxation and classic results like Chamley Judd [or Straub Werning (2019)’s reassessment].

I strongly encourage you to review Stancheva’s capital tax lecture slides:
1. **Corporate Income Tax** (fed+state): 21% Federal tax rate on profits of corporations [complex rules with many industry specific provisions]: effective tax rate lower.

2. **Individual Income Tax** (fed+state): taxes many forms of capital income
   - Realized capital gains and dividends receive preferential treatment (to lower double taxation of corporate profits)
   - Imputed rent of home owners and returns on pension funds are exempt

3. **Estate tax:** tax on very large estates (40% tax above $11m) bequeathed to heirs (now very small and poorly enforced)

4. **Property taxes** (local) on real estate (old tax):
   - Tax varies across jurisdictions. About 0.5% of market value on average
   - Won’t be able to discuss land taxation or housing subsidies in detail, but big deal/important area [see Henry George's Progress and Poverty, which sold millions of copies (second only to Bible in 1890s) and helped spark Progressive Era].
Proposals to reform current taxation of wealth and capital income

1. **Wealth**
   - Estate tax on inheritances
   - Local property tax

2. **Capital income**
   - Corporate tax
   - Individual income tax

But some cite concerns:
- Estate tax avoidance concerns, property tax not very progressive
- Low corporate tax rate (21%) and lack of integration ⇒ Rich will incorporate and accumulate within corporations
- Realized capital gains tax partly retained earnings and pure K gains but with loopholes (deferral and step-up of basis after transfer/inheritance)
Recall estimated progressivity of US tax system in 2018

Source: Saez Zucman (2019)
Recall estimated progressivity of US tax system in 1962

Source: Saez Zucman (2019)
Democrats’ Emerging Tax Idea: Look Beyond Income, Target Wealth

Lawmakers and 2020 candidates offer a range of options focused on capturing some of the trillions of dollars in assets belonging to the nation’s richest
A related proposal of accrual taxation

- **Mark-to-market:** tax gains as they accrue. Assets valued every year, and taxpayers pay taxes on the gain or deduct the loss.

- **Retroactive accrual:** tax gains upon sale. Minimize benefit of deferring sale by including deferral charge equivalent to back taxes due with interest.

- **Combination approach:** mark-to-market for publicly traded assets and retroactive accrual for non-publicly traded assets (harder to price annually).
Proposed accrual tax plans

- **Sen. Ron Wyden**
  - Combination approach: mark-to-market and retroactive accrual
  - Applied only to top earners (≥ $1 million in annual income) and top wealth-holders (≥ $10 million in assets for three consecutive years, with some exemptions)
  - Use ordinary-income tax rates, no specified top rate
  - Use revenues to fund Social Security

- **Joe Biden**
  - Tax unrealized gains at death, abolishing stepped-up basis
  - Double income-tax rate on capital gains (currently 20%) for taxpayers with income ≥ $1 million
  - Revenues delayed relative to other plans
A range of proposals
Note that these plans treat “buy, borrow, die” strategy differently

### Rethinking Capital Gains Taxation

Democrats are looking at major changes to the way capital gains taxation works. The effects of their tax proposals would depend on each taxpayer’s circumstances and on market performance.

<table>
<thead>
<tr>
<th>CURRENT LAW</th>
<th>BIDEN PLAN</th>
<th>WYDEN PLAN</th>
<th>WARREN PLAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets can appreciate without capital gains taxes and heirs pay taxes only on gains in value after the original owner’s death.</td>
<td>Death would be considered a realization event, triggering capital gains taxes on appreciated assets, paid at ordinary income tax rates.</td>
<td>Each year, investors would pay income taxes on the gain in their assets. This is called a mark-to-market system.</td>
<td>Net worth above $50 million subject to a 2% annual tax, plus a 1% tax on net worth above $1 billion.</td>
</tr>
</tbody>
</table>

Example one: Asset value begins at $40 million, 5% growth until person dies in year 25

Total taxes taken under law/plans:

- $0
- $35.6 million
- $21.4 million
- $11.9 million

Example two: Asset value begins at $200 million, 7% growth until year 12, person dies in year 25

Total taxes taken under law/plans:

- $0
- $88.4 million
- $58.3 million
- $121.2 million

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Impact of a Capital Tax
Impact of a Capital Tax

![Graph showing the impact of a capital tax](image)
Impact of a Capital Tax (in Long Run)

\[ R_t \]

\[ R^* \]

\[ R_{\text{pre-tax}} \]

\[ R_{\text{post-tax}} \]

\[ K^* \]

\[ \text{Long-Run } S(R) \]

\[ D(R_t) \]
Impact of a Capital Tax

Who bears the capital tax in the long run?

- Who gets the triangle above R-pre-tax (i.e., consumer surplus in the typical S and D graph)?
- If firms don’t earn profits, this all goes to workers in terms of higher wages or lower prices
- A key object is the *elasticity of capital supply*, is likely larger (and some think infinite) in the LR
- Note that the distortion in the capital market reduces surplus more than it increases tax revenues (as with most taxes)
- Finally, distortions due to capital taxation are often considered in a dynamic context in which the distortion compounds overtime (See Werning Straub (2019) on the classic Chamley-Judd results)
Preliminaries: use and asset markets for capital
We will use 4 equations to analyze capital markets

1. **Stock Adjustment**: the amount of capital today depends on how much there was yesterday, depreciation, and new investment.

2. **Asset pricing equilibrium**: the rental price of using an asset is simply the cost of buying the good and re-selling it after one period.

3. **Rental market equilibrium**: the demand for using capital services is downward sloping.

4. **Investment market equilibrium**: the supply of capital assets is upward sloping.

Rental and asset markets are linked

Use the link between rental and asset markets to analyze capital markets

\[ R_t \] is the rental price of using capital services \( K_t \) and \( P_t \) is the purchase price, which depends on the level of investment \( l_t \).

where \( R_t \) is the **rental price** of using capital services \( K_t \) and \( P_t \) is the **purchase price**, which depends on the level of investment \( l_t \).
4 key equations

1. **Stock Adjustment:** \( K_t = (1 - \delta)K_{t-1} + I_t \)

2. **Asset pricing equilibrium** The rental cost of using an asset is simply the cost of buying the good and re-selling it after one period.

3. **Rental market equilibrium:** \( K = D(R) \)

4. **Investment market equilibrium:** \( I = S(P) \)
What is the relationship between rental and capital prices?

The rental cost of using an asset is simply the cost of buying the good and re-selling it after one period

\[ R_t = P_t - \frac{(1 - \delta)P_{t+1}}{1 + r} \]

- \( r \) is the nominal rate of interest
- \( P_{t+1} \) is next year’s price for the good
Suppose

- Suppose \( r = 0.10 \) and \( \delta = 0 \)
- \( P_{t+1} = \$110 \text{ K} \)
- \( P_t = \$100 \text{ K} \)
- What is \( R_t \)?

\[
R_t = P_t - (1 - \delta) P_{t+1} + \frac{1}{1 + r} \]

\[
R_t = 100 - \frac{110}{1.10} = 0
\]
2. Asset pricing equilibrium: Housing example

Suppose

- Suppose \( r = .10 \) and \( \delta = 0 \)
- \( P_{t+1} = $110 \) K
- \( P_t = $100 \) K
- What is \( R_t \)?

\[
R_t = P_t - \frac{(1 - \delta)P_{t+1}}{1 + r}
\]

\[
R_t = 100 - \frac{110}{1 + .1}
\]

\[
R_t = 0
\]
We can rearrange the expression to show rental prices depend on three things:

\[ R_t = \frac{rP_t + \delta P_{t+1} + P_t - P_{t+1}}{1 + r} \]

1. Interest cost\(^3\): \(rP_t\)
2. Depreciation: \(\delta P_{t+1}\)
3. Market re-evaluation: \(P_t - P_{t+1}\)

Rental prices are higher, the higher is \(r\), the greater is the physical rate of depreciation, and the faster the price of the asset is declining.
2. Analyzing Rental Price: Car example

\[ R_t = \frac{rP_t + \delta P_{t+1} + P_t - P_{t+1}}{1 + r} \]

- If cars lose their value quickly (i.e., \( P_t >> P_{t+1} \)), then rental prices will be pretty high.
We can also use the rental price expression to calculate the implied capital price

\[
P_t = R_t + \frac{R_{t+1}(1 - \delta)}{(1 + r)} + \frac{R_{t+2}(1 - \delta)^2}{(1 + r)^2} + \ldots
\]

- This equation can be obtained by recursively substituting for future prices in the rental price equation
- This equation should look familiar to you (prices are PV of cash flow stream)
- Capital prices are higher when rental payments to the owner are large and soon
3. Rental Market Equilibrium for Housing Services

\[ K_t = D(R_t) \]

- The demand for housing services depends on the flow cost of housing services (i.e., the rental rate \( R_t \)). \( R_t \) is what I pay to use the asset.
- Housing services are provided by the stock of housing \( K_t \).
- The demand side of the market links the current rental price and the current stock.
3. Rental Market Equilibrium

\[ R_t \]

\[ K_t = D_t(R_t) \]

\[ D_t(R_t) \]
4. Investment Market Equilibrium

\[ I_t = S(P_t) \]

- The supply of new construction, investment depends on its current price.
- Think of this as a new car producer who decides how much to supply based on the current price.
- Alternatively, housing construction firms see high house prices and build. They build more when prices are high.
4. Investment Market Equilibrium
4 key equations

\[ K_t = (1 - \delta)K_{t-1} + I_t \]  \hspace{1cm} (1)

\[ R_t = P_t - \frac{(1 - \delta)P_{t+1}}{1 + r} \]  \hspace{1cm} (2)

\[ K_t = D(R_t) \]  \hspace{1cm} (3)

\[ I_t = I(P_t) \]  \hspace{1cm} (4)

4 equations and 4 unknowns, but depends on past and the future. Where do past and future come in?
When we look at a market equilibrium for the housing market at any one point in time, we must realize that today’s market is influenced by both the past and future. The effect of the past comes through the effect of past production decisions on the stock of housing. The effect of the future comes from the effect of future expected rental rates on the current price.
What does the system look like in steady state?

\[
\begin{align*}
\bar{K} &= (1 - \delta)\bar{K} + \bar{I} \\
\bar{R} &= \bar{P} - \frac{(1 - \delta)\bar{P}}{1 + r} \\
\bar{K} &= D(\bar{R}) \\
\bar{I} &= S(\bar{P})
\end{align*}
\]
What does the system look like in steady state?

\[
\bar{I} = \delta \bar{K} \\
\bar{R} = \bar{P} \left( 1 - \frac{1 - \delta}{1 + r} \right) \\
\bar{K} = D(\bar{R}) \\
\bar{I} = S(\bar{P})
\]
What does the system look like in steady state?

We can use the first two equations to plug into the second two equations and obtain the supply and demand in the use market.

\[
\bar{I} = \delta \bar{K} \\
\frac{\bar{R}}{\left(1 - \frac{(1-\delta)}{1+r}\right)} = \bar{P} \\
\bar{K} = D(\bar{R}) \\
\bar{I} = S(\delta \bar{K}, \frac{\bar{R}}{\left(1 - \frac{(1-\delta)}{1+r}\right)})
\]
What does the system look like in steady state?

\[ \tilde{K} = D(\tilde{R}) \]

\[ \tilde{K} = \frac{1}{\delta} S \left( \frac{\tilde{R}}{\left(1 - \frac{(1-\delta)}{1+r}\right)} \right) \]

This shows that we have a familiar supply and demand diagram where the quantity is \( K \) and the price is \( R \).
Capital Market Equilibrium

\[ R_t \]

\[ K_t \]

\[ K^* \]

\[ R^* \]

\[ D(R_t) \]

\[ S(R_t) \]
Earthquake Destroys part of capital stock
The main impact is on the use market. Lower $K$ increases $R$.
Higher rental prices cause the asset price $P$ to increase.
However, since rental rates decline as we rebuild capital stock, the increase in $P$ is smaller than increase in $R$.
Investment follows $P$, so it will jump and slowly decline as we rebuild the stock.
Earthquake Destroys part of capital stock
What determines the speed of convergence to the steady state?

1. **Elasticity of demand** in the rental market $\varepsilon^D$. For example, the more the rental price goes up following a destruction of the capital stock, the faster we will converge to steady state (since it will make the capital price go up more, and thereby also investments). With a higher elasticity (in absolute value), the rental price will go up more.

2. **Elasticity of supply** in the investment market $\varepsilon^S$. This will make investment go up more when the capital price goes up.

3. The **depreciation rate** $\delta$. This may be the most important aspect, since it puts a lower bound on the speed of convergence. The slowest rate at which the economy ever can return to the steady state is $\delta$.

Others examples: construction costs, interest rates, housing bubble?
Continuous-time versions of the 4 key equations

\[ \dot{K}(t) = -\delta K(t) + I(t) \]  
\[ P(t) = \int_t^\infty e^{-(r+\delta)(z-t)} R(z) \, dz \]  
\[ K(t) = D(R(t)) \]  
\[ I(t) = I(P(t)) \]

Sometimes we use \( R(t) = (r + \delta)P(t) - \dot{P}(t) \), which is implied by the second expression. Note, that this expression, however, does not imply the 2nd expression b/c it doesn’t include a boundary condition that says prices must be set at the correct level.
Combining equations

\[ \dot{K}(t) = -\delta K(t) + I(P(t)) \]

\[ K(t) = D((r + \delta)P(t) - \dot{P}(t)) \]
Phase diagram

- Above the $\dot{K}(t) = 0$ line, the price is high, so the capital stock is increasing (so arrow above $\dot{K}(t) = 0$ points to the right).
- To the right of the $\dot{P}(t) = 0$ line, the level of capital stock is too high, so the price is too low and is therefore rising overtime (so we have an up arrow).
- Convergence to the steady state happens along the saddle path.

See https://home.uchicago.edu/cbm4/cpt/videos.html chapter 16 for more discussion.
Taxes, Housing, and Asset Markets (Poterba, 1984)
Q: what is the effect of changes in inflation and tax policy on housing market (prices and quantities)

The paper develops and estimates an asset-market model of the US housing market
- Starts with the demand for housing services as a function of tax policy and inflation
- Then solves for asset prices
- Then adds supply side to incorporate how quantities change when asset prices change
- Calibrates demand side and estimates supply side and then runs simulations/counterfactuals

Finds as much as 30 percent of the increase in the real price of owner-occupied structures could be attributed to the user cost decline of the late 1970s

I’ll present a related model/framework from Poterba’s AEA slides (notation is a bit different than 1984 paper)
Equilibrium in the use market for housing services

For a single period $t$, equilibrium in the housing services market is defined as:

$$R(H_t) = q_{H,t}(r(1 - \tau) + \delta) - (q_{H,t+1} - q_{H,t})$$  \hspace{1cm} (9)$$

- $R(H_t)$: Value of asset services per unit of housing
- $q_{H,t}$: House price at start of period $t$
- $q_{H,t+1} - q_{H,t}$: Capital gain or loss during period $t$
- $r(1 - \tau)$: Opportunity cost of funds (i.e., after-tax interest rate)
- $\tau$: Marginal income tax rate
- $\delta$: Depreciation

Source: Jim Poterba, 2013: Public Economics AEA slides
Add in a tax

If a tax of required payment $T$ is levied on houses in period $t$, equilibrium becomes:

$$R(H_t) - T_t = q_{H,t}(r(1 - \tau) + \delta) - (q_{H,t+1} - q_{H,t})$$

$$= q_{H,t}(1 + r(1 - \tau) + \delta) - q_{H,t+1}$$

We can rewrite this as

$$q_{H,t} = \frac{R(H_t) - T_t + q_{H,t+1}}{1 + r(1 - \tau) + \delta}$$
Solve for house prices (1/2)

We assume perfect foresight so that we can solve by substituting recursively for $q_{H,t+1}$:

\[
q_{H,t} = \left( \sum_{t=0}^{S} \frac{R(H_{t+i}) - T_{t+i}}{(1 + r(1 - \tau) + \delta)^{i+1}} \right) + \frac{q_{H,t+S}}{(1 + r(1 - \tau) + \delta)^{S+1}}
\]  

(13)

We also impose a transversality condition so that the house price won’t ”explode” (and so we can drop this term):

\[
\lim_{S \to \infty} \frac{q_{H,t+S}}{(1 + r(1 - \tau) + \delta)^{S+1}} = 0
\]
For \( S = \infty \), we can rewrite our expression for \( q_{H,t} \):

\[
q_{H,t} = \sum_{t=0}^{\infty} \frac{R(H_{t+i})}{(1 + r(1 - \tau) + \delta)^{i+1}} - \sum_{t=0}^{\infty} \frac{T_{t+i}}{(1 + r(1 - \tau) + \delta)^{i+1}}
\]

Present discounted value of current and future tax payments

(14)
Effect on house prices depends on supply side

What does this model tell us about changes in house prices over time? Two possibilities:

- **Housing stock is fixed:** We can determine $\frac{\Delta q_{H,t}}{\Delta T_{t+1}}$ from equation 14.
- **Housing stock is endogenous:** higher taxes $\rightarrow$ lower house prices $\rightarrow$ decline in housing construction $\rightarrow$ decreased future stock of housing capital $\rightarrow$ rise in house prices

To formalize the latter case, we must model the supply function for new construction.
Supply side: new home construction increases in house prices

For $I_t$ denoting the gross construction of new housing, we declare

$$I_t = \Psi(q_{H,t})$$  \hspace{1cm} (15)

So, the net change in the housing stock over a period is given by:

$$H_{t+1} - H_t = \Psi(q_{H,t}) - \delta H_t$$  \hspace{1cm} (16)

We can rearrange 10 to see that the corresponding change in house prices over a period is:

$$q_{H,t+1} - q_{H,t} = q_{H,t}(r(1 - \tau) + \delta) - R(H_t) + T_t$$  \hspace{1cm} (17)
Taxes and housing market dynamics

Equations 16 and 17 show that the steady state is defined by two equations:

\[ \Psi(q_H) = \delta H \tag{18} \]
\[ q_H(r(1 - \tau) + \delta) = R(H) - T \tag{19} \]

We can graph this steady state along with the impact of a tax increase:
Applying a more realistic user cost framework

To illustrate how this framework can be used to evaluate changes to housing tax policy (pre-TCJA), consider the user cost of owner-occupied housing:

\[ c = \left(1 - \tau_y\right)\left(\gamma r_M + (1 - \gamma) r_{Alt} + \beta\right) + m + (1 - \tau_y) \tau_{prop} - \pi_e \]  

\[ \tau_y = \text{marginal inc. tax rate for mortgage int. and prop. tax deducts, invest. inc.} \]
\[ \tau_{prop} = \text{property tax rate} \]
\[ \gamma = \text{home’s loan-to-value ratio} \]
\[ r_M = \text{mortgage interest rate} \]
\[ r_{Alt} = \text{return on non-house assets} \]
\[ m = \text{depreciation + maintenance costs} \]
\[ \beta = \text{Pre-tax housing risk premium} \]
\[ \pi_e = \text{expected rate of nominal house price appreciation} \]
Calibrating user costs and effects of tax changes

For 2012, some plausible parameters are

\[ \tau_y = 0.25 \quad r_M = r_{Alt} = 0.04 \]
\[ \tau_{prop} = 0.015 \quad m = \beta = 0.02 \]
\[ \gamma = 0.75 \quad \pi_e = 0.02 \]

This implies a user cost \( c \) of 0.056. Eliminating the federal income tax deduction for the property tax would increase \( c \) to 0.06.

How would this affect house prices?

<table>
<thead>
<tr>
<th>Model</th>
<th>Implied ( \Delta ) housing prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant housing stock, elasticity of demand of -1</td>
<td>-6.7%</td>
</tr>
<tr>
<td>Endogenous stock, calibrated rational expectations model</td>
<td>-3.7%</td>
</tr>
</tbody>
</table>
Simple spatial model: One factor, two locations
Impact of Capital Tax: One factor, two locations

Setup

1. One factor (capital)
2. Two locations: east and west
3. Capital market in each location
4. Total $K$ fixed in economy overall
Initial equilibrium

Diagram showing the relationship between interest rate ($r$) and capital ($K$) with a point $K_0$ where $r = r_0$. The graph illustrates the initial equilibrium in the market.
Causes capital to flee to east

\[ r \]

\[ K_1 \quad K_0 \]

\[ r_0 \]

\[ K_0 \quad K_1 \]
New allocation of capital

- $K$ flows to east, lowering net returns in both
- Flows continue until after tax return is equalized across markets
Welfare changes in each location

- Welfare in west falls by red amount
- Welfare in east increases
Net welfare changes in aggregate

- Net welfare loss in red

\[ r_{\text{gross}} \]
\[ r_0 \]
\[ r_{\text{net}} \]

\[ K_1 \quad K_0 \]

\[ K_0 \quad K_1 \]
What determines size of welfare loss in this toy example?

1. Size of tax change
2. Size of market being taxed (depends on fundamentals)
3. Elasticity of demand in both regions (quantity response more generally, which depends on S and D elasticities)
4. Strength of complementarities across markets (e.g., labor market)
5. Assumptions about effects/value of government spending (assumed to be zero here)
6. Presence of existing distortions

Will formalize these ideas in the next section, but this example provides intuition for key forces in the Harberger model.
Outline

1 Brief Introduction
   - What is capital income?
   - Sources of wealth and capital income
   - How is capital income taxed in the US?

2 Capital Tax Incidence: simple models and asset prices
   - Supply and demand in the capital market
   - Preliminaries rental and asset markets for capital
   - Application: Taxes, Housing, and Asset Markets (Poterba, 1984)
   - Simple spatial model: One factor, two locations

3 Capital Tax Incidence: Harberger
   - Fullerton and Ta (2017)
     - Welfare loss from taxation
     - Effect of Tax on Corporate Output
     - Effect of Tax on Capital
     - Effect of Tax on Corporate Capital
   - Harberger Model (more general utility and technology)
Overview

Goals
- Characterize effects of corporate tax change in a GE model
- Who bears the burden of corporate taxes? (also capital, output taxes)

Two sectors (or locations)
- Corporate sector produces output $X$
- Non-corporate sector produces output $Y$

Markets
- Capital: prices $r_i$, quantities $K_i$ where $i \in \{X, Y\}$
- Labor: prices $w_i$, quantities $L_i$
- Goods: prices $p_i$, quantities $X, Y$

Agents
- Workers (representative, perfectly mobile, supply 1 unit of labor)
- Firm (representative, perfectly competitive, CRS)

Equilibrium Conditions
- Good and factor markets clear, factor price equalization
- Consumers max utility, firms earn zero profits
Harberger is workhorse analytical model: 2 sector and 2 factors
Fixed supply of capital and labor (short run, closed economy)
Key intuition is misallocation (magnitude depends on factor intensity, demand elasticities, etc)
Fullerton and Ta (2017) simplifies Harberger analysis (Cobb Douglas)
Similar to Hecksher-Ohlin model
When interpreting as locations not sectors, then implicitly assume no trade costs. Similarly, implicitly assumes no adjustment costs for capital and labor (so long run in that sense)
Abstracts from amenity or productivity effects of government spending (lump sum rebates or purchases in same share as consumers)
Fullerton and Ta (2017)
Parameterized Harberger Model with Cobb Douglas
Consumers: Preferences and Budget Constraint

Utility of representative worker is $U = X^\gamma Y^{1-\gamma}$
- $X$ is corporate sector output
- $Y$ is non-corporate sector output

Budget constraint is $p_x X + p_y Y = I$
- $I$ is income, which is sum of labor and capital income
- $p_i$ is price of output in sector $i$ where $i \in \{X, Y\}$

Workers have fixed expenditure shares (e.g. $I\gamma$); demand for $X$ and $Y$ is:

$$X = \frac{I\gamma}{p_x}$$

$$Y = \frac{I(1-\gamma)}{p_y}$$

N.B. note no labor supply or saving decision
Indirect utility is given

\[ V(p_x, p_y, I) = \left( \frac{I \gamma}{p_x} \right) \gamma \left( \frac{I(1 - \gamma)}{p_y} \right)^{1-\gamma} = \frac{I}{\bar{p}} \]

where \( \bar{p} = \left( \frac{p_x}{\gamma} \right)^\gamma \left( \frac{p_y}{1-\gamma} \right)^{1-\gamma} \) is the “ideal” price index

Inverting indirect utility (i.e., \( V = \frac{I}{\bar{p}} \)), gives the expenditure function \( I = E \):

\[ E(\bar{p}, U) = U\bar{p} \]

So \( \bar{p} \) is the price paid for each “util”
Firms maximize profits

- Corporate sector solves:

  \[ \max_{K_x, L_x} (1 - \tau_X)p_x X - (1 + \tau_K + \tau_{KX})rK_x - wL_x, \text{ where } X = AK_x^{\alpha}L_x^{1-\alpha} \]

- where

  \[ \tau_X = \text{ tax on output of } X \]
  \[ \tau_K = \text{ tax on capital} \]
  \[ \tau_{KX} = \text{ tax on capital in production of } X \]

- Non-corporate sector solves:

  \[ \max_{K_y, L_y} p_y Y - (1 + \tau_K)rK_y - wL_y, \text{ where } Y = BK_y^{\beta}L_y^{1-\beta} \]
Firm optimization (and factor demand)

FOCs:

\[ w = (1 - \tau_X)p_x(1 - \alpha)A \left( \frac{K_x}{L_x} \right)^\alpha \]

\[ w = p_y(1 - \beta)B \left( \frac{K_y}{L_y} \right)^\beta \]

and

\[ (1 + \tau_K + \tau_{KX})r = (1 - \tau_X)p_x\alpha A \left( \frac{L_x}{K_x} \right)^{1-\alpha} \]

\[ (1 + \tau_K)r = p_y\beta B \left( \frac{L_y}{K_y} \right)^{1-\beta} \]
Exogenous parameters

- **Taxes**: $\tau_X$, $\tau_K$, $\tau_{KX}$
  - $\tau_X$ is tax on corporate sector output (sales tax)
  - $\tau_K$ is tax on capital
  - $\tau_{KX}$ is tax on capital used in corporate sector

- **Consumer Parameter**: $\gamma$
  - $\gamma$ governs importance of corporate goods for utility
  - $1 - \gamma$ governs importance of non-corporate goods for utility

- **Firm Parameters**: $\alpha, \beta, A, B$
  - $\alpha$ is output elasticity of capital in sector $X$
  - $1 - \alpha$ output elasticity of labor in sector $X$
  - $\beta$ output elasticity of capital in sector $Y$
  - $1 - \beta$ output elasticity of labor in sector $Y$
  - $A$ and $B$ are productivity in corp and non-corp sectors

- **Endowments**: $K, L$
  - $K$ is total capital
  - $L$ is total labor
Endogenous outcomes are $K_i, L_i, p_i, X, Y, w, r$:

- **Capital**: prices $r_i$, quantities $K_i$ where $i \in \{X, Y\}$
- **Labor**: prices $w_i$, quantities $L_i$
- **Goods**: prices $p_i$, quantities $X, Y$

*Given $\tau_X, \tau_K, \tau_{KX}, \gamma, \alpha, \beta, A, B, K, L$, equilibrium is defined by prices and quantities \{w, r, p_i, K_X, K_Y, L_X, L_Y, X, Y\} such that good and factor markets clear and firms and workers optimize.*
Equilibrium: closed form expressions
In log terms, the equations are:

\[
\begin{align*}
\ln X &= \ln l + \ln \gamma - \ln p_x \\
\ln Y &= \ln l + \ln(1 - \gamma) - \ln p_y \\
\ln K_x - \ln L_x - \ln w + \ln r &= \ln \alpha - \ln(1 - \alpha) - \ln(1 + \tau_K + \tau_{KX}) \\
\ln K_y - \ln L_y - \ln w + \ln r &= \ln \beta - \ln(1 - \beta) - \ln(1 + \tau_K) \\
\ln X - \ln K_x + \ln p_x - \ln r &= \ln(1 + \tau_K + \tau_{KX}) - \ln \alpha - \ln(1 - \tau_X) \\
- \ln X + \ln L_x - \ln p_x + \ln w &= \ln(1 - \alpha) + \ln(1 - \tau_X) \\
\ln Y - \ln K_y + \ln p_y - \ln r &= \ln(1 + \tau_K) - \ln \beta \\
- \ln Y + \ln K_y - \ln p_y + \ln w &= \ln(1 - \beta)
\end{align*}
\]

where \( K = K_x + K_y \) and \( L = L_x + L_y \).
Given taxes $\tau_K$, $\tau_X$, and $\tau_{KX}$, we have

$$X = A \gamma (1 - \tau_X) \left( \frac{\alpha (1 + \tau_K)}{\alpha \gamma (1 - \tau_X)(1 + \tau_K) + \beta (1 - \gamma)(1 + \tau_K + \tau_{KX})} K \right)^\alpha \left( \frac{(1 - \alpha)}{(1 - \alpha) \gamma (1 - \tau_X) + (1 - \beta)(1 - \gamma) L} \right)^{1-\alpha}$$

$$Y = B (1 - \gamma) \left( \frac{\beta (1 + \tau_K + \tau_{KX})}{\alpha \gamma (1 - \tau_X)(1 + \tau_K) + \beta (1 - \gamma)(1 + \tau_K + \tau_{KX})} K \right)^\beta \left( \frac{(1 - \beta)}{(1 - \alpha) \gamma (1 - \tau_X) + (1 - \beta)(1 - \gamma) L} \right)^{1-\beta}$$

$$K_x = \frac{\alpha \gamma (1 - \tau_X)(1 + \tau_K)}{\alpha \gamma (1 - \tau_X)(1 + \tau_K) + \beta (1 - \gamma)(1 + \tau_K + \tau_{KX})} K$$

$$K_y = \frac{\beta (1 - \gamma)(1 + \tau_K + \tau_{KX})}{\alpha \gamma (1 - \tau_X)(1 + \tau_K) + \beta (1 - \gamma)(1 + \tau_K + \tau_{KX})} K$$

$$L_x = \frac{(1 - \alpha) \gamma (1 - \tau_X)}{(1 - \alpha) \gamma (1 - \tau_X) + (1 - \beta)(1 - \gamma)} L$$
\begin{align*}
L_y &= \frac{(1 - \beta)(1 - \gamma)}{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)} L \\
p_x &= \frac{l}{A(1 - \tau_X)} \left( \frac{\alpha \gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_KX)}{\alpha(1 + \tau_K)K} \right)^\alpha \\
&\quad \times \left( \frac{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)}{(1 - \alpha)L} \right)^{1-\alpha} \\
p_y &= \frac{l}{B} \left( \frac{\alpha \gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_KX)}{\beta(1 + \tau_K + \tau_KX)K} \right)^\beta \\
&\quad \times \left( \frac{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)}{(1 - \beta)L} \right)^{1-\beta} \\
w &= \frac{l}{L} \left[ (1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma) \right] \\
r &= \frac{l}{K} \left[ \frac{\alpha \gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_KX)}{(1 + \tau_K)(1 + \tau_K + \tau_KX)} \right]
\end{align*}
Welfare Loss from Taxation
Equivalent variation and Burden of taxation

- Equivalent variation \( EV \) is the change in wealth at initial prices that would be equivalent to the price change in terms of utility.

\[
EV = E(p^0_1, U^1) - I
\]

\[
= p^0_1 U^1 - p^0_0 U^0 = p^0_0 (U^1 - U^0)
\]

where \( p^0_0 \) and \( p^1 \) are the “ideal” prices in period 0 and 1

- Use \(-EV\) as a positive measure of tax burden, so

\[
EB = -EV = p^0_0 (U^0 - U^1)
\]

Amount that burden exceeds tax revenues is called excess burden (Auerbach and Hines, 2002). We consider a revenue-neutral reform with a distorting tax where all revenue is returned lump sum, so net revenue is zero and thus, net loss is excess burden.
Average and Marginal Excess Burden

- **Average Excess Burden (AEB)** is the total welfare loss from the tax divided by the total revenue collected by the government:

  \[
  AEB = \frac{EB}{R}
  \]

  where \( \bar{p}^0 \) and \( \bar{p}^1 \) are the “ideal” prices in period 0 and 1

- **Marginal excess burden (MEB)** measures the effects of a small change in the tax rate on burden:

  \[
  \Delta EB = \bar{p}^0(EB^1 - EB^2)
  \]

  \[
  MEB = \frac{\Delta EB}{\Delta R}
  \]

N.B. See Hendren’s recent TPE paper for more detailed discussion
Understanding Equilibrium: Graphical and quantitative analysis
There are a lot of moving parts

- Helpful to think about relative factor markets (relative prices and relative quantities) in the two sectors
- Will start with demand side, then supply side, then analyze equilibrium graphically pre and post taxes
- Will work with a calibrated version of the model to do quantitative analysis
Relative factor demand

Taking ratios of each sector’s FOCs gives:

\[
\frac{w}{r} = \frac{(1 - \alpha)}{\alpha} \left( \frac{L_x}{K_x} \right)^{-1} (1 + \tau_K + \tau_{KX}) \tag{21}
\]

\[
\frac{w}{r} = \frac{(1 - \beta)}{\beta} \left( \frac{L_y}{K_y} \right)^{-1} (1 + \tau_K) \tag{22}
\]
Relative factor supply

Recall

\[ L = L_x + L_y \]
\[ K = K_x + K_y \]

Thus, the economy-wide labor capital ratio is:

\[ \frac{L}{K} = \frac{L_x}{K} + \frac{L_y}{K} \]

\[ \frac{L}{K} = \frac{L_x}{K_x} \left( \frac{K_x}{K} \right) + \frac{L_y}{K_y} \left( \frac{K_y}{K} \right) \]  (23)

This says that overall labor to capital ratio is a weighted average of the labor to capital ratio in both sectors
We can invert 21 and 22 to get \( L_x/K_x \) and \( L_y/K_y \) as functions of \( w/r \). Then

\[
\frac{L}{K} = \left( \frac{w}{r} \right)^{-1} \left( \frac{(1 + \tau_k + \tau_{kx})(1 - \alpha)}{\alpha} \frac{K_x}{K} + \frac{(1 + \tau_k)(1 - \beta)}{\beta} \frac{K_y}{K} \right)
\]

(24)

In equilibrium, we found

\[
\frac{K_x}{K} = \frac{\alpha \gamma (1 - \tau_x)(1 + \tau_K)}{\alpha \gamma (1 - \tau_x)(1 + \tau_K) + \beta (1 - \gamma)(1 + \tau_K + \tau_{kx})}
\]

(25)

\[
\frac{K_y}{K} = \frac{\beta (1 - \gamma)(1 + \tau_K + \tau_{kx})}{\alpha \gamma (1 - \tau_x)(1 + \tau_K) + \beta (1 - \gamma)(1 + \tau_K + \tau_{kx})}
\]

(26)
Numerical Example

- \(X\) and \(Y\) produced given functions

\[
X = AK_x^6 L_x^4 \\
Y = BK_y^2 L_y^8
\]

- Identical households have utility:

\[
U = X^{0.5} Y^{0.5}
\]

- Fixed\(^4\) level of income \(I = 2,400\)

- Demand for \(X\) and \(Y\) is given by:

\[
X = \frac{2400(0.5)}{p_x} = \frac{1200}{p_x} \\
Y = \frac{2400(0.5)}{p_y} = \frac{1200}{p_y}
\]

- Assume unity of prices in the initial state \((p_x = p_y = r = w = 1)\)
Initial quantities and prices ($\tau_K = \tau_{KX} = \tau_X = 0$)

- With this parameterization of utility and technology, we have:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_x$</td>
<td>480</td>
</tr>
<tr>
<td>$L_y$</td>
<td>960</td>
</tr>
<tr>
<td>$K_x$</td>
<td>720</td>
</tr>
<tr>
<td>$K_y$</td>
<td>240</td>
</tr>
<tr>
<td>$X$</td>
<td>1200</td>
</tr>
<tr>
<td>$Y$</td>
<td>1200</td>
</tr>
<tr>
<td>$p_x$</td>
<td>1</td>
</tr>
<tr>
<td>$p_y$</td>
<td>1</td>
</tr>
<tr>
<td>$r$</td>
<td>1</td>
</tr>
<tr>
<td>$w$</td>
<td>1</td>
</tr>
</tbody>
</table>

- Use the values above to derive $A \approx 1.96$ and $B \approx 1.69$ (given output, inputs, and production function)
Initial Factor Market Equilibrium \( (\tau_K = \tau_{KX} = \tau_X = 0) \)

\[
\begin{align*}
\tau_K & = \tau_{KX} = \tau_X = 0 \\
\text{Figure: Wage to Rent Ratio in both sectors and economy overall}
\end{align*}
\]
Effect of Tax on Corporate Output ($\tau_X = .3$)

1. $\tau_X$ reduces demand for $X$

2. We will have factors move from producing $X$ to producing $Y$ until prices and quantities re-equilibrate

3. Specifically, since $w_x = w_y \Rightarrow (1 + \tau_x) p_x MPL_x = p_y MPL_y$, we need a combination of lower $p_x$ and higher $MPL_x$ (and thus lower factor demand in $x$) and/or higher $p_y$ and lower $MPL_y$

4. The movement of both factors to $Y$ increases the weight of the non-corporate sector in labor and capital demand (see eq 24 and dashed green line in next slide)

5. Since the non-corporate sector is relatively labor intensive, total relative labor demand increases

6. Hence, the equilibrium wage to rental ratio increases
Tax on Corporate Output

Figure: Relative Factor market equilibrium with $\tau_X = 0.3$, $\tau_K = \tau_KX = 0$.
### Panel A: Allocations and Prices

<table>
<thead>
<tr>
<th></th>
<th>$t_X = 0$</th>
<th>$t_X = 0.3$</th>
<th>$t_X = 0.31$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_x$</td>
<td>480</td>
<td>373.333</td>
<td>369.368</td>
</tr>
<tr>
<td>$L_y$</td>
<td>960</td>
<td>1,066.667</td>
<td>1,070.632</td>
</tr>
<tr>
<td>$K_x$</td>
<td>720</td>
<td>650.323</td>
<td>647.296</td>
</tr>
<tr>
<td>$K_y$</td>
<td>240</td>
<td>309.677</td>
<td>312.704</td>
</tr>
<tr>
<td>$X$</td>
<td>1,200</td>
<td>1,020.942</td>
<td>1,013.750</td>
</tr>
<tr>
<td>$Y$</td>
<td>1,200</td>
<td>1,373.811</td>
<td>1,380.577</td>
</tr>
<tr>
<td>$p_x$</td>
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<tr>
<td>$p_y$</td>
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<tr>
<td>$w$</td>
<td>1</td>
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<td>$w/r$</td>
<td>1</td>
<td>1.161</td>
<td>1.168</td>
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<tr>
<td>$L_x/K_x$</td>
<td>0.667</td>
<td>0.574</td>
<td>0.571</td>
</tr>
<tr>
<td>$L_y/K_y$</td>
<td>4</td>
<td>3.444</td>
<td>3.424</td>
</tr>
<tr>
<td>$L/K$</td>
<td>1.500</td>
<td>1.500</td>
<td>1.500</td>
</tr>
</tbody>
</table>
### Panel B: Exact Measures of Welfare

<table>
<thead>
<tr>
<th></th>
<th>( t_X = 0 )</th>
<th>( t_X = 0.3 )</th>
<th>( t_X = 0.31 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{p}^0 )</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \bar{p} )</td>
<td>2</td>
<td>2.026</td>
<td>2.029</td>
</tr>
<tr>
<td>( U )</td>
<td>1,200</td>
<td>1,184.306</td>
<td>1,183.030</td>
</tr>
<tr>
<td>( EB )</td>
<td>0</td>
<td>31.387</td>
<td>33.940</td>
</tr>
<tr>
<td>( R )</td>
<td>0</td>
<td>360</td>
<td>372</td>
</tr>
<tr>
<td>( AEB )</td>
<td>0.087</td>
<td>0.091</td>
<td></td>
</tr>
<tr>
<td>( MEB )</td>
<td>0.213</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Suppose a tax on all capital: $\tau_K = 0.3$, and $\tau_{KX} = \tau_X = 0$

Both sectors face tax on capital, so capital allocation across sectors does not change (see 25 and 26 in which the $(1 + \tau_k)$ terms cancel)

$(1 + \tau_k)$ increases relative labor demand symmetrically in eq 24 in both sectors (i.e., it shifts up $L_i/K_i$), so factor allocation stays constant and all adjustment is through relative prices

In this case, capital fully bears the burden of the tax (i.e., $w/r$ rises by 30% to offset tax increase)

N.B. remember that in these examples, the overall stock of capital is fixed. In practice, investment and firm creation respond to taxes. A key question is how much they respond
Tax on Capital

Figure: Relative Factor market equilibrium with $\tau_K = 0.3$, $\tau_{KX} = \tau_X = 0$.
### Panel A: Allocations and Prices

<table>
<thead>
<tr>
<th></th>
<th>$t_K=0$</th>
<th>$t_K=.3$</th>
<th>$t_K=.31$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_x$</td>
<td>480</td>
<td>480</td>
<td>480</td>
</tr>
<tr>
<td>$L_y$</td>
<td>960</td>
<td>960</td>
<td>960</td>
</tr>
<tr>
<td>$K_x$</td>
<td>720</td>
<td>720</td>
<td>720</td>
</tr>
<tr>
<td>$K_y$</td>
<td>240</td>
<td>240</td>
<td>240</td>
</tr>
<tr>
<td>$X$</td>
<td>1,200</td>
<td>1,200</td>
<td>1,200</td>
</tr>
<tr>
<td>$Y$</td>
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<tr>
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<tr>
<td>$p_y$</td>
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<td>1</td>
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<tr>
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<td>$w$</td>
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<td>1</td>
</tr>
<tr>
<td>$w/r$</td>
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### Panel B: Exact Measures of Welfare

<table>
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Effect of Tax on Corporate Capital

1. Now suppose a tax on corporate capital, $\tau_{KX} = .3$

2. Corporate sector demands less capital ($r_x = \frac{p_x MPK_x}{1 + \tau_{KX}}$), so capital flows from corporate to non-corporate sector (see eq 25 and 26)

3. Lower capital allocation to producing $X$ increases the weight of the non-corporate sector in labor and capital demand (see eq 24 and dashed green line in next slide)

4. Causes misallocation (too much $K_y$ and thus, too much $Y$, not enough $X$), which reduces welfare as in prior example
Figure: Relative factor market equilibrium with $\tau_{LX} = 0.3$, $\tau_{LX} = \tau_{KX} = 0$.
### Panel A: Allocations and Prices

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### Panel B: Exact Measures of Welfare

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<tr>
<td>$MEB$</td>
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Harberger Model
Harberger (more general utility and technology)

Ten equations needed for equilibrium are:

\[ p_x X + p_y Y = wL + rK \]  \quad (27)

\[ MRS_{XY} = \frac{p_x (1 + \tau_X)}{p_y} \]  \quad (28)

\[ c_x(w, r(1 + \tau_K + \tau_{KX})) = p_x \]  \quad (29)

\[ c_y(w, r(1 + \tau_K)) = p_y \]  \quad (30)

\[ w = p_x F_{xL} \]  \quad (31)

\[ w = p_y F_{yL} \]  \quad (32)

\[ r(1 + \tau_K + \tau_{KX}) = p_x (1 - \tau_X) F_{xK} \]  \quad (33)

\[ r = p_y F_{yK} \]  \quad (34)

\[ K = K_x + K_y \]  \quad (35)

\[ L = L_x + L_y \]  \quad (36)
Definitions

- Share of income spent on $X$ and $Y$:
  \[ s_x \equiv \frac{p_x X}{p_x X + p_y Y}, \quad s_y \equiv \frac{p_y Y}{p_x X + p_y Y}, \quad s_x + s_y = 1 \]

- Share of income from labor and capital:
  \[ s_w \equiv \frac{wL}{wL + rK}, \quad s_r \equiv \frac{rK}{wL + rK} \]

- Cost shares in production of $X$ and $Y$:
  \[ \theta_L \equiv \frac{wL_x}{wL_x + rK_x}, \quad \theta_K \equiv \frac{rK_x}{wL_x + rK_x}, \quad \theta_x + \theta_y = 1 \]
  \[ \phi_L \equiv \frac{wL_y}{wL_y + rK_y}, \quad \phi_K \equiv \frac{rK_y}{wL_y + rK_y}, \quad \phi_x + \phi_y = 1 \]
Definitions

- Share of labor and capital used to produce $X$:
  \[
  \lambda_L \equiv \frac{L_x}{L}, \quad \lambda_K \equiv \frac{K_x}{K}
  \]

- By Euler’s Theorem and CRS, we also have:
  
  \[
  p_x X = wL_x + rK_x, \quad p_y Y = wL_y + rK_y
  \]
  \[
  \Rightarrow \lambda_L = \frac{s_x \theta_L}{s_x \theta_L + s_y \phi_L} = \frac{s_x(1 - \theta_K)}{1 - s_x \theta_K - s_y \phi_K}
  \]
  \[
  \Rightarrow \lambda_K = \frac{s_x \theta_K}{s_x \theta_K + s_y \phi_K} = \frac{s_x(1 - \theta_L)}{1 - s_x \theta_L - s_y \phi_L}
  \]
\[ s_x(\hat{p}_x + \hat{X}) + s_y(\hat{p}_y + \hat{Y}) = s_w\hat{w} + s_r\hat{R} \]

\[ \hat{X} - \hat{Y} = \sigma_D(\hat{p}_y - \hat{p}_x - d\tau_X) \]

\[ \hat{p}_x = \theta_L\hat{w} + \theta_K(\hat{r} + d\tau_K + d\tau_KX) \]

\[ \hat{p}_y = \phi_L\hat{w} + \phi_K(\hat{r} + d\tau_K) \]

\[ \lambda_L\hat{L}_x + (1 - \lambda_L)\hat{L}_y = 0 \]

\[ \lambda_K\hat{K}_x + (1 - \lambda_K)\hat{K}_y = 0 \]

\[ \hat{L}_x = \hat{X} + \theta_K\sigma_X(\hat{r} + d\tau_K + d\tau_KX - \hat{w}) \]

\[ \hat{K}_x = \hat{X} + \theta_L\sigma_X(\hat{w} - d\tau_K - d\tau_KX - \hat{r}) \]

\[ \hat{L}_y = \hat{Y} + \phi_K\sigma_Y(\hat{r} + d\tau_K - \hat{w}) \]

\[ \hat{K}_y = \hat{Y} + \phi_L\sigma_Y(\hat{w} - d\tau_K - \hat{r}) \]
Matrix Form of the System of Linear Equations

\[
\begin{bmatrix}
\sigma_D & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\
1 & -\theta_L & -\theta_K & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\phi_L & -\phi_K & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_L & 0 & 1 - \lambda_L & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_K & 0 & 1 - \lambda_K & 0 \\
0 & \theta_K \sigma_X & -\theta_K \sigma_X & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & -\theta_L \sigma_X & \theta_L \sigma_X & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & \phi_K \sigma_Y & -\phi_K \sigma_Y & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & -\phi_L \sigma_Y & \phi_L \sigma_Y & 0 & -1 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\hat{p}_x \\
\hat{w} \\
\hat{r} \\
\hat{X} \\
\hat{Y} \\
\hat{L}_x \\
\hat{K}_x \\
\hat{L}_y \\
\hat{K}_y \\
\end{bmatrix}
= 
\begin{bmatrix}
-\sigma_X \\
-\theta_K \\
-\phi_K \\
\phi_K \sigma_X \\
\theta_K \sigma_X \\
\theta_L \sigma_X \\
\phi_K \sigma_Y \\
-\phi_K \sigma_Y \\
\end{bmatrix}
d\tau +
\begin{bmatrix}
0 \\
-\theta_K \\
0 \\
\theta_K \sigma_X \\
0 \\
\theta_L \sigma_X \\
0 \\
0 \\
\end{bmatrix}
d\tau_{KX}
\]

where we eliminated equation 1 by Walras law and normalized \( p_y = 1 \), so \( \hat{p}_y = 0 \)
Two Main Effects of Taxing $K_x$

1. **Substitution effects:** capital bears incidence

2. **Output effects:** capital may not bear all incidence
Substitution effects

- Tax on $K_x$ shifts production in $X$ away from $K$ so aggregate demand for $K$ goes down
- Because total $K$ is fixed, $r$ falls $\rightarrow K$ bears some of the burden

Another intuition for this is that capital is misallocated across sectors, which lowers $r$ and $rK$
Output effects

- Tax on $K_x$ makes $X$ more expensive
- Demand shifts to $Y$
- **Case 1:** $K_x/L_x > K_y/L_y$ ($X$: cars, $Y$: bikes)
  - $X$ more capital intensive $\rightarrow$ lower aggregate demand for $K$
  - Output + subst. effect: $K$ bears the burden of the tax
- **Case 2:** $K_x/L_x < K_y/L_y$ ($X$: bikes, $Y$: cars)
  - $X$ less capital intensive $\rightarrow$ higher aggregate demand for $K$
  - Subst. and output effects have opposite signs $\rightarrow$ labor may bear some of the tax
Harberger showed that under a variety of reasonable assumptions, capital bears exactly 100 percent of the tax. Note that this is the burden on all capital – as capital flees the corporate sector, it depresses returns in the noncorporate sector as well. Both the realism of the model and the characterization of the corporate income tax as an extra tax on capital in the corporate sector are subject to question, as discussed in considerable detail by the subsequent literature on the effects of the corporate tax. – Alan Auerbach

See Auerbach TPE paper on who bears the corporate tax for more details.
Appendix:
Ten equations and ten unknowns (without taxes)

In log terms, the equations are:

\[ \ln X = \ln A + \alpha \ln K_x + (1 - \alpha) \ln L_x \]
\[ = \ln I + \ln \gamma - \ln p_x \]

\[ \ln Y = \ln B + \beta \ln K_y + (1 - \beta) \ln L_y \]
\[ = \ln I + \ln (1 - \gamma) - \ln p_y \]

\[ \ln w = \ln p_x + \ln (1 - \alpha) + \ln A + \alpha (\ln K_x - \ln L_x) \]
\[ = \ln p_y + \ln (1 - \beta) + \ln B + \beta (\ln K_y - \ln L_y) \]

\[ \ln r = \ln p_x + \ln \alpha + \ln A + (1 - \alpha)(\ln L_x - \ln K_x) \]
\[ = \ln p_y + \ln \beta + \ln B + (1 - \beta)(\ln L_y - \ln K_y) \]

where \( K = K_x + K_y \) and \( L = L_x + L_y \).
Solutions in initial equilibrium without taxes (1/2)

- Solving for the system of equations, quantities are:

\[
X = A\gamma \left( \frac{\alpha K}{\alpha \gamma + \beta(1 - \gamma)} \right)^\alpha \left( \frac{(1 - \alpha)L}{(1 - \alpha)\gamma + (1 - \beta)(1 - \gamma)} \right)^{1-\alpha}
\]

\[
Y = B(1 - \gamma) \left( \frac{\beta K}{\alpha \gamma + \beta(1 - \gamma)} \right)^\beta \left( \frac{(1 - \beta)L}{(1 - \alpha)\gamma + (1 - \beta)(1 - \gamma)} \right)^{1-\beta}
\]

\[
K_x = \frac{\alpha \gamma}{\alpha \gamma + \beta(1 - \gamma)} K
\]

\[
K_y = \frac{\beta(1 - \gamma)}{\alpha \gamma + \beta(1 - \gamma)} K
\]

\[
L_x = \frac{(1 - \alpha)\gamma}{(1 - \alpha)\gamma + (1 - \beta)(1 - \gamma)} L
\]

\[
L_y = \frac{(1 - \beta)(1 - \gamma)}{(1 - \alpha)\gamma + (1 - \beta)(1 - \gamma)} L
\]
Prices are:

\[ p_x = \frac{I}{A} \left( \frac{\alpha \gamma + \beta (1 - \gamma)}{\alpha K} \right)^\alpha \left( \frac{(1 - \alpha)\gamma + (1 - \beta)(1 - \gamma)}{(1 - \alpha)L} \right)^{1-\alpha} \]

\[ p_y = \frac{I}{B} \left( \frac{\alpha \gamma + \beta (1 - \gamma)}{\beta K} \right)^\beta \left( \frac{(1 - \alpha)\gamma + (1 - \beta)(1 - \gamma)}{(1 - \beta)L} \right)^{1-\beta} \]

\[ w = \frac{I}{L} \left( (1 - \alpha)\gamma + (1 - \beta)(1 - \gamma) \right) \]

\[ r = \frac{I}{K} \left( \alpha \gamma + \beta (1 - \gamma) \right) \]
Suppose the economy is in steady state and suddenly the costs of new construction increase.
Suppose the economy is in steady state and suddenly the costs of new construction increase.

- The main impact is on the asset market. Supply curve shifts left.
- Lower supply decreases new construction
- The lower stock of capital causes rents to rise
- Rising rents and the prospects of future higher rents (remember long-run rents are higher) cause the price to rise
- As prices rise over time, construction rebounds
Increase the cost of new construction
Suppose the economy is in steady state and suddenly the interest rate $r$ declines.

Lower $r \Rightarrow$ future streams of payments are more valuable, so $P$ will jump up. The jump in asset prices causes investment to jump up. More investment increases the capital stock. Higher capital stocks start to decrease rental rates. Higher rental rates decrease asset prices. We will end at a steady state with higher $K^*$ and lower $R^*$. 

Graduate Public Finance (Econ 524)
Decline in the interest rate

Suppose the economy is in steady state and suddenly the interest rate $r$ declines.

- Lower $r \Rightarrow$ future streams of payments are more valuable, so $P$ will jump up
- The jump in asset prices causes investment to jump up
- More investment increases the capital stock
- Higher capital stocks start to decrease rental rates
- Higher rental rates decrease asset prices
- We will end at a steady state with higher $K^*$ and lower $R^*$
Decline in the interest rate

\[ R, P, I, K \text{ at } t=0 \]
US House Prices and Residential Investment

- Real Private Residential Fixed Investment, 2001:Q1=100
- All-Transactions House Price Index for the United States, 2001:Q1=100

Graph showing trends in real private residential fixed investment and all-transactions house price index from 2002 to 2014.
Was this an irrational bubble?
What did we see in the housing boom in the 2000s?

Low interest rates and high capital prices. Therefore, housing services are cheap to use and investment will increase as $P$ increases with $P$. Suppose people expected higher future demand. More downward pressure on rental prices, higher capital prices, more construction.

Graduate Public Finance (Econ 524)

Capital Taxes

Lecture 5a
What did we see in the **housing boom** in the 2000s?

- Low interest rates and high capital prices
- Therefore, housing services are cheap to use and investment will increase as $S(P)$ increases with $P$
- Suppose people expected higher future demand
- More downward pressure on rental prices, higher capital prices, more construction
What did we see in the **housing bust** in the late 2000s?

We've build up a large housing stock. Suppose now the anticipated increase in demand never comes:

- Falling house prices
- Big decline in investment
- Also makes consuming housing services more expensive
- Things will adjust as housing stock goes back to the steady state

Was this an irrational bubble?

Observing a crash in the housing market does not tell us whether (1) there were rational expectations about future demand (coupled with low interest rates) or (2) a bubble (that could not be justified by expectations).
What did we see in the **housing bust** in the late 2000s?

- We’ve build up a large housing stock
- Suppose now the anticipated increase in demand never comes
  - $\Rightarrow$ Falling house prices
  - $\Rightarrow$ Big decline in investment
  - $\Rightarrow$ Also makes consuming housing services more expensive
  - $\Rightarrow$ Things will adjust as housing stock goes back to the steady state

**Was this an irrational bubble?**

Observing a crash in the housing market does not tell us whether (1) there were rational expectations about future demand (coupled with low interest rates) or (2) a bubble (that could not be justified by expectations).