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Princeton
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Lecture 2
1 Background

2 Model
   - Overview
   - Workers: marginal worker is indifferent between locations
   - Landlords: have upward sloping housing supply
   - Firm: makes traded good, zero profits
   - Gov’t finances wage subsidy with lump-sum tax

3 Equilibrium
   - Comparative Statics: Graphical Results
   - Comparative Statics: Analytical Results

4 Welfare Effects
   - Welfare Comparative Statics: Graphical Results
   - Winners and Losers

5 Other considerations: market imperfections, second-best, and equity
Outline

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Motivation

- Substantial differences in incomes across locations
  - Wages in Stamford, CT is 2X same worker in Jacksonville, NC
  - In 2009, unemployment rate in Flint, MI was 6X that of Iowa City, Iowa

- These differences persist across decades and generations

- Lucas “I don’t see how one can look at figures like these without seeing them as possibilities”

- Many governments institute development policies aimed at increasing growth in lagging areas and reducing spatial disparities within their location
Questions

- How large are place-based policies?
- Who benefits from place-based policies?
- Do the national benefits outweigh the costs?
- What types of interventions are most likely to be effective?
State and Local economic development spending
A substantial portion is for local business incentives. Source Barik (2019)

Table 3 Resources Devoted to State and Local Economic Development in the United States

<table>
<thead>
<tr>
<th>CURRENT PROGRAMS</th>
<th>Annual dollars (in billions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Policy/program</td>
<td></td>
</tr>
<tr>
<td>State and local business tax incentives and other cash incentives</td>
<td>46.3</td>
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<tr>
<td>Customized training programs</td>
<td>0.6</td>
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<tr>
<td>Other state economic development programs</td>
<td>2.8</td>
</tr>
<tr>
<td><strong>Subtotal, state/local programs</strong></td>
<td><strong>49.7</strong></td>
</tr>
<tr>
<td>Manufacturing extension (federal/state/fees)</td>
<td>0.4</td>
</tr>
<tr>
<td>Economic Development Administration (EDA)</td>
<td>0.3</td>
</tr>
<tr>
<td>Economic development portion of HUD’s Community Development Block Grants</td>
<td>1.1</td>
</tr>
<tr>
<td>Small Business Administration</td>
<td>0.8</td>
</tr>
<tr>
<td>Other economic development programs in USDA, HUD, Commerce</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>Subtotal, mostly federal spending</strong></td>
<td><strong>4.6</strong></td>
</tr>
<tr>
<td>Opportunity Zones tax credits</td>
<td>1.5</td>
</tr>
<tr>
<td>New markets tax credit</td>
<td>1.4</td>
</tr>
<tr>
<td>Other tax expenditures that might promote local economic development</td>
<td>2.3</td>
</tr>
<tr>
<td><strong>Subtotal, federal tax expenditures</strong></td>
<td><strong>5.3</strong></td>
</tr>
<tr>
<td><strong>Total of federal programs and tax expenditures</strong></td>
<td><strong>9.9</strong></td>
</tr>
<tr>
<td><strong>Total of all levels of government</strong></td>
<td><strong>59.6</strong></td>
</tr>
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</table>

<table>
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<tr>
<th>PAST PROGRAMS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Empowerment zones/enterprise communities (peak annual activity in early 2000s)</td>
<td>1.5</td>
</tr>
<tr>
<td>Appalachian Regional Commission (peak annual spending 1966–1975)</td>
<td>1.6</td>
</tr>
<tr>
<td>Tennessee Valley Authority (peak annual spending 1950–1955)</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Source: Tim Barik (2019)
Poor states receive a lot of federal aid

Per capita spending and revenues vs per capita income

Source: Paul Krugman (2019)
Poor states receive a lot of federal aid

Source: Paul Krugman (2019)
Rationales for place-based policies

- **Equity**
  1. Economists have generally been skeptical of equity-based arguments, as location is being used to serve a person-based motive: subsidizing poor households (see Glaser and Gottlieb, 2008)
  2. Could do so more directly through tax progressive or transfer programs
  3. Mobility can undermine spatial targeting. Rosen-roback model (with mobile workers and inelastic housing supply) predicts that entire benefit of location-based subsidies will be capitalized into land rents
  4. However, if workers (or firms) are less mobile, redistributive policies can benefit inframarginal workers (firms)

- **Efficiency:** Can remedy market failures
  1. Public Goods (amenities like public safety or productive public goods like roads)
  2. Agglomeration
  3. Labor market frictions
  4. Missing insurance/ credit markets
  5. Pre-existing distortions
A recent case for place-based policy

Larry Summers at Boston Fed, Oct 2019

1. There’s widely uneven incidence of distress in US in terms of employment, opportunity, health, etc
2. National economic forces are doing little to cause this problem to solve itself
3. The propensity for migration to take place has diminished quite substantially and outmigrants are likely to be most able, skilled, catalytic
4. Disaffection of non-cosmopolitans who live away from major prosperous cities is a key source of protectionism, nationalism, anti-globalization, etc
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Overview

1. Goals
   - Characterize effect of place-based wage subsidy on prices (wages and rents), city size, and welfare
   - Determine aggregate benefits (costs) and how they are distributed across agents and locations

2. Two Locations $c \in \{a, b\}$

3. Markets
   - Local labor and housing: price $w_c$, quantity $N_c$. Price $r_c, N_c$
   - Global capital and goods: price $\rho$, quantity $K_c$. Price $p = 1, Y_c$

4. Agents
   - Workers (continuum, have heterogeneous taste draws)
   - Landlord (representative, housing has upward sloping supply)
   - Firm (perfectly competitive, CRS, traded good)
   - Government provides ad valorem wage credit $\tau_c$ to firms

5. Key Indifference Condition
   - Marginal worker has same indirect utility in both locations
Indirect utility of individual $i$ in location $c$ is given

$$U_{ic} = w_c - r_c + A_c - t + e_{ic} \equiv v_c$$

where

- nominal wages $w_c$
- cost of housing $r_c$
- lump sum taxes $t$
- local amenities $A_c$
- common indirect utility component $v_c$
- $e_{ic}$ represents worker $i$’s idiosyncratic preferences for location $c$
Workers: Idiosyncratic Component of Indirect Utility

- $e_{ic}$ are i.i.d. according to a Type I Extreme Value distribution with scale parameter $s$ and mean 0

$$\frac{e_{ia} - e_{ib}}{s} \sim \text{logistic}(0, 1)$$

- $s$ governs the **strength** of idiosyncratic preferences for location, i.e., the degree of labor mobility

  - if $s$ is:
    - *large*, then many workers will need large real-wage or amenity differences to move
    - *small*, then most workers will move in response to small real-wage or amenity differences
    - *0*, then workers will arbitrage any differences in the systematic component of utility (Rosen-Roback baseline)
Worker location decision determines local labor supply

- Workers choose the location that maximizes their utility
- A worker chooses city $a$ if and only if
  \[ e_{ib} - e_{ia} < v_{ia} - v_{ib} \]
- The fraction of workers locating in city $a$ can be expressed as:
  \[ N_a = \Lambda \left( \frac{v_a - v_b}{s} \right) \]
  where $\Lambda(\cdot) = \frac{\exp(\cdot)}{1 + \exp(\cdot)}$ is the standard logistic cumulative density function
- The number of workers residing in community $a$ is increasing in:
  - the *real-wage gap* between city $a$ and city $b$, $(w_a - r_a) - (w_b - r_b)$
  - the *difference in amenities* between the cities, $A_a - A_b$
**Big picture**: $s$ and the $\Lambda(\cdot)$ distribution are a way of getting upward sloping labor supply and having inframarginal workers who can benefit from local policies.

- Logistic distribution is not essential. Many trade folks like Frechet.
- Indirect utility is linear in $r_c$, which implies each person uses a house but has no intensive margin response when wages increase.
- If preferences are Cobb-Douglas over housing and non-housing as in Suarez-Serrato and Zidar (AER 2016), you’ll get an expression for indirect utility that is log linear and implies that expenditure shares will be fixed (so higher income means you spend more on housing).
- Diamond (AER 2016) models endogenous amenities $A_c(N_c)$ that are increasing with population.
Elasticity of Local Labor Supply depends on $s$

- The elasticity of city size with respect to city-specific components of utility:

$$\frac{d \ln N_a}{d \ln (v_a - v_b)} = \frac{N_b}{s} (v_a - v_b)$$

- This elasticity varies based on the intensity of preferences for location:
  - if $s$ is small, then workers are very sensitive to differences in mean utility between cities
  - if $s$ is 0, the any real-wage difference not offset by a corresponding difference in amenities results in the entire population of workers choosing the location with the higher mean utility
Landlords: Housing supply is stylized, upward sloping

- Housing is supplied competitively (note: it requires no workers)
- Land is fixed, so the marginal cost of housing is increasing in the number of units produced
- Constant elasticity\(^2\) inverse supply function:

\[
    r_c = z_c N_c^{k_c}
\]

where \(N_c\) (number of workers in location \(c\)) is assumed to be equal to the number of housing units in location \(c\)
- \(z_c\) governs housing productivity (lower \(z_c\) increases supply of housing)
- \(k_c\) governs the elasticity of housing supply
- \(k_c\) is determined by geography and land regulations, and it is:
  - \textit{small} in cities where geography and regulations make it easy to build new housing
  - \(0\) in locations where there are no constraints to building new houses
Representative firm makes traded good, zero profits

- Firms produce a single good $Y$ using labor and a local amenity
- $Y$ is a traded good sold on international markets at price 1
- Cobb-Douglas production function with constant returns to scale:

$$Y_c = X_c N_c^\alpha K_c^{1-\alpha}$$

where:
- $X_c$ is a city-specific productivity shifter
- $N_c$ is the fraction of workers in community $c$
- $K_c$ is the local capital stock
- Firms can rent as much capital as desired at fixed price $\rho$
The government provides an ad valorem wage credit $\tau_c$ to employers in community $c$.

Lump sum taxes are levied on all workers in both locations to finance the wage credit.

Balanced budget constraint:

$$w_a\tau_a N_a + w_b\tau_b N_b = t$$

Firms equate the marginal revenue product of labor to wages net of taxes:

$$w_c(1 - \tau_c) = \alpha \frac{y_c}{N_c}$$

First-order condition for capital:

$$\rho = (1 - \alpha) \frac{y_c}{K_c}$$
Inverse labor demand schedule in location $c$:

$$\ln w_c = C + \frac{\ln X_c}{\alpha} - \frac{1 - \alpha}{\alpha} \ln \rho - \ln(1 - \tau_c)$$

where $C \equiv \ln \alpha + \frac{1 - \alpha}{\alpha} \ln(1 - \alpha)$

inverse labor demand is horizontal in the wage-employment space due to:

- production function with constant returns to scale
- elastic supply of capital at price $\rho$

wage variation across cities stems from variation in productivity levels

firms make zero profits (so can’t bear incidence. See SS-Z AER, 2016)
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Local Labor Market Equilibrium

- Equilibrium: the marginal worker’s relative preference for city $b$ over city $a$ equals the difference in real wages net of amenities:

$$s\Lambda^{-1}(N_a) = (w_a - w_b) - (r_a - r_b) + (A_a - A_b)$$

- Workers whose relative preference for city $b$ is greater (smaller) than the real-wage gap net of amenities locate in city $b$ ($a$)

- City size is ultimately determined by fundamentals:

$$s\Lambda^{-1}(N_a) = \frac{e^C}{\rho} \left( \frac{X_a^{\frac{1}{\alpha}}}{1 - \tau_a} - \frac{X_b^{\frac{1}{\alpha}}}{1 - \tau_b} \right) +$$

$$- \left( z_a N_a^k - z_b (1 - N_a)^k b \right) + A=a - A_b$$

Where:
- $s\Lambda^{-1}(N_a)$ represents the relative preference
- $N_a$ is the population of city $a$
- $w_a$ and $w_b$ are the real wages in cities $a$ and $b$
- $r_a$ and $r_b$ are the real returns to capital in cities $a$ and $b$
- $A_a$ and $A_b$ are the amenities in cities $a$ and $b$

Taste Differences
- Wage difference
- Rent difference
- Amenity difference
Local Labor Market Equilibrium

- LHS: quantiles of workers’ relative preferences \( (e_{ib} - e_{ia}) \) for city \( b \) as a function of \( N_a \) \( \Rightarrow \) supply curve to city \( a \)
- RHS: difference in mean utilities between the two communities \( \Rightarrow \) relative demand curve for residence in city \( a \) vs. city \( b \)
- Equilibrium at the intersection of the two curves:
  - A *single marginal worker* is indifferent between city \( a \) and city \( b \)
  - All other workers are *inframarginal* and enjoy a strictly positive consumer surplus associated with residing in the city they strictly prefer
A Two-City Model: Equilibrium Comparative Statics

![Graph showing the relationship between utility and the fraction of population in city $a$ with different tax rates.](image)
Ceteribus paribus, the degree of labor mobility increases ($s \downarrow$)
Ceteribus paribus, the housing price elasticity in location A increases \((k_A \uparrow)\)
A Two-City Model: Labor Effects

- An increase in the wage subsidy in city $a$ yields an increase in the nominal wage in $a$:
  \[
  \frac{d w_a}{d \tau_a} = \frac{w_a}{1 - \tau_a}
  \]

- Workers in city $b$ are unaffected by an increase in the wage subsidy to workers in city $a$.

- $N_a$ increases because some workers move from $a$ to $b$:
  \[
  \frac{d N_a}{d \tau_a} = \frac{N_a N_b}{s + k_b r_b N_a + k_a r_a N_b} \frac{w_a}{1 - \tau_a}
  \]

- The number of movers is larger:
  - the smaller is $s$, which implies that labor is more mobile in response to real-wage differentials.
  - the larger is the elasticity of housing supply in city $a$ (i.e., the smaller is $k_a$), which implies that it is easier for city $a$ to add new housing units to accommodate the increased demand.
A Two-City Model: Housing Market Effects

- An increase in the wage subsidy in city $a$ yields an increase in the cost of housing in $a$:

$$\frac{dr_a}{d\tau_a} = \frac{k_a r_a N_b}{s + k_b r_b N_a + k_a r_a N_b} \frac{w_a}{1 - \tau_a}$$

- Conversely, the cost of housing decreases in city $b$:

$$\frac{dr_b}{d\tau_a} = \frac{k_b r_b N_a}{s + k_b r_b N_a + k_a r_a N_b} \frac{w_a}{1 - \tau_a}$$

- The increase in $r_a$ is increasing in $k_a$
- The decrease in $r_b$ is increasing in $k_b$
A Two-City Model: Real Wage Effects

- An increase in the wage subsidy in city a yields an economywide increase in real wages.
- In community a:
  \[
  \frac{d(w_a - r_a)}{d\tau_a} = \frac{s + k_b r_b N_a}{s + k_b r_b N_a + k_a r_a N_b} \frac{w_a}{1 - \tau_a} > 0
  \]

- In community b:
  - nominal wages are unaffected
  - the cost of housing falls
  thus leading to higher real wages

- The reason why real wages increase in both cities differs:
  - city a: the subsidy raises nominal wages more than housing costs
  - city b: workers out-migrate

- The real-wage increase in city a is larger than the increase in city b, unless labor is perfectly mobile \((s = 0)\), in which case the increase is the same.
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Worker welfare is defined as the average utility level given optimal location choices:

\[ V = E \max\{ U_{ia}, U_{ib} \} = s \log \left( \exp \left( \frac{v_a}{s} \right) + \exp \left( \frac{v_b}{s} \right) \right) \]

An increase in the subsidy to community \( a \) yields:

\[ \frac{dV}{d\tau_a} = N_a \frac{d(w_a - r_a)}{d\tau_a} + N_b \frac{d(w_b - r_b)}{d\tau_a} - \frac{dt}{d\tau_a} \]

The impact of a subsidy to city \( a \) equals:

- the impact on real wages in \( a \) times the share of workers in \( a \), plus
- the impact on real wages in \( b \) times the share of workers in \( b \), minus
- the cost of raising funds

Movers do not show up in this expression because they were indifferent about the communities to begin with.
A Two-City Model: Gains and Losses

The diagram illustrates the utility function for different scenarios in two cities, A and B, with varying tax rates. The graph shows the utility for different levels of parameter $\Lambda (e_{lb} - e_{la}) / S$, with distinct lines representing utility in city A and city B under different tax scenarios. The gains and losses are indicated for original residents of city A, gains and losses for movers, and losses for stayers in city B.
Efficiency Costs

- Average worker utility
  
  \[ V = E \max \{ U_{ia}, U_{ib} \} = s \log \left( \exp \left( \frac{v_a}{s} \right) + \exp \left( \frac{v_b}{s} \right) \right) \]

- For small subsidy, impact on welfare is impact on after-tax disposable income
  
  \[ \frac{dV}{d \tau_a} = N_a \frac{d(w_a - r_a)}{d \tau_a} + N_b \frac{d(w_b - r_b)}{d \tau_a} - \frac{dt}{d \tau_a} \]

- Net impact on worker utility + landlord profits
  
  \[ \frac{d(V + \Pi_a + \Pi_b)}{d \tau_a} = -\eta N_a \tau_a \]

  where \( \eta = -\frac{dN_a}{d(1-\tau_a)} \frac{1-\tau_a}{N_a} \geq 0 \) gives mobility elasticity

- Harberger (1964) “triangle” approximation: \( DWL \approx \frac{1}{2} \eta \tau_a^2 N_a \)
Subsidizing a place yields a transfer to targeted households (and landlords) but distorts location decisions.

Efficient transfer: no quantity response / job creation!

Ramsey (1927)-style targeting principle: subsidize locations that are least elastic.

Empirical question: when are elasticities big?
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Market imperfections and additional considerations

See Kline and Moretti, Annual Review 2014 for discussion

- Local public goods
  - (Suárez Serrato and Wingender, 2014)
- Agglomeration Economies \( \ln X_c = g(\text{density}_c, HC_c) \)
  - Big push (Kline, 2013) and (Kline Moretti 2014 on TVA)
- Unemployment, Labor and Product Market Frictions
  - Hiring costs (Kline and Moretti, 2013), (Bilal 2019)
  - Keynesian frictions in spatial models (Rodrguez-Clare, 2020)
- Credit Constraints and Missing Insurance
  - Location as an asset (Bilal Rossi-Hansberg, 2019)
- Many other second best considerations ...
Correct prior distortions that can interact with place

- Deductibility of state and local taxes (Albouy, 2009)
- State sales & biz taxes (Fajgelbaum, Morales, Serrato, Zidar, 2019)
- Housing Regulations (Hsieh and Moretti, 2019)
- Intergovernmental transfers (Albouy, 2012)
- Payroll taxes?
- Subsidy war as prisoner’s dilemma (Ossa, 2019)
- Transportation Infrastructure (Donaldson, 2020)
- Allocation of talent (Gaubert Fajgelbaum, 2019) vs (Moretti, 2019), (Rossi-Hansberg, Sarte, Schwartzman, 2019)
Equity considerations

- Place conveys useful information about preferences and endowments
- Odd to ignore when setting policy
- In “Place-Based Redistribution,” Gaubert, Kline, & Yagan (2019) study whether place-based transfers to individuals can still improve welfare in a world with an optimal income tax
- Answer turns out to be yes when there is either strong skill taste correlation or strong income effects in location.
Not obvious that PBR would be desirable!

"Place-Based Redistribution" by Gaubert, Kline, & Yagan

- Urban intuition appears to rule this out
  - Key assumption: perfect mobility/location indifference
  - PBR causes people to move to less productive places
  - Why would we want to increase activity in less productive areas?

- Public Finance intuition also seems to rule this out
  - Key assumption: preferences are weakly separable and homogeneous
  - Notorious Atkinson-Stiglitz result: an optimal income tax can take care of all forms of redistribution
Why do they find PBR is desirable?

“Place-Based Redistribution” by Gaubert, Kline, & Yagan

Urban Side: Assume people have location-specific preferences/imperfect mobility

PF Side: Show weak separability does not apply in this case
  - True when tastes for amenities vary by income and when there is income sorting
  - When high earners sort elsewhere, equity motive for spatial targeting to distressed areas

With two main roadblock out of the way ... off to the races of optimal taxation!
What the Paper Does
“Place-Based Redistribution” by Gaubert, Kline, & Yagan

- Shows when the introduction of small PBR is desirable in special cases
  - Skill-taste correlation
  - Sorting through income effects
  - Productivity differences

- General results:
  - Introducing small PBR is desirable if value of redistribution outweighs fiscal cost of productivity differences from migration
  - Optimal PBR depends on further migration effects that also have fiscal costs

- Quantitative exercise finds a small PBR to bottom CZs can improve welfare
General model shows PBR is desirable when:

\[ \lambda_1 - \lambda_0 > \mathbb{E}_\theta \left\{ \frac{dS^\theta(0)}{d\Delta} \left[ T(z_0^\theta) - T(z_1^\theta) \right] \right\} \]

(1) Gain from redistribution

(2) Induced Migration

(3) Tax cost of migration

Paper shows:

1. can be assumed to be positive
2. migration only matters due to the fiscal externality
3. cost of tax loss depends on productivity differences
Appendix: Review of Discrete Choice
Aside on Discrete Choice

- Brief review of discrete choice
- CDF of tastes and demand curves
- Link to demand elasticities
- See Ken Train’s *Discrete Choice Methods with Simulation* (free online) for very clear, helpful discussion
Consumers decide whether or not to buy

Fraction Purchasing
Consumers decide whether or not to buy.
Consumers decide whether or not to buy

- The first graph shows the share of consumers buying a product is 50% when its price is $5
- The second graph shows the share of consumers buying a product is 30% when its price is $6
- How can we think about how responsive demand will be to changes in price when consumers are making discrete (i.e., buy or not) choices?
Suppose that individual $i$ buys if her value exceeds the price, i.e., buy if $v_i > P$.

This value can be a function of common things (e.g., income, credit conditions, etc) or idiosyncratic tastes but at this stage, specifying what is in $v_i$ doesn’t matter. The fraction of people who buy is:

$$\text{Prob}(Q = 1) = P(v_i > P)$$  \hspace{1cm} (2)

$$= 1 - F(P)$$  \hspace{1cm} (3)

where $F(x)$ is the c.d.f. of $v_i$. Note this is why the demand curve looks like a CDF rotated clockwise 90 degrees.

A c.d.f. describes the probability that a real-valued random variable $X$ with a given probability distribution will be found to have a value less than or equal to $x$. 

Elasticity of Demand

What is the elasticity of this curve?

$$Q(P) = N(1 - F(P))$$  \hspace{1cm} (4)

where \( N \) is the size of the population (e.g., number of potential consumers in your market)

$$\varepsilon^D = \frac{dQ(P)}{dP} \frac{Q}{P}$$  \hspace{1cm} (5)
Elasticity of Demand

What is the derivative?

\[
\frac{dQ(P)}{dP} = -Nf(P)
\]  

(6)

- where \( N \) is the size of the population (e.g., first time home buyers in an area)
- \( f(x) \) is the probability density function (p.d.f.)
Elasticity of Demand

\[ \varepsilon^D = \frac{dQ(P)}{dP} \frac{P}{Q} \]  

(7)

\[ = -Nf(P) \frac{P}{N(1 - F(P))} \]  

(8)

\[ = -f(P) \frac{1}{1 - F(P)} P \]  

(9)

What matters for responsiveness?

- Fraction of people at the margin \( f(P) \)
- Fraction of people already buying \( 1 - F(P) \)
From $5, a $1 dollar increase in price ↓ demand by 20%
From $8, a $1 dollar increase in price ↓ demand by 2%
Elasticity of Demand: In words

Takeaways:

- For very homogeneous populations, you’ll have very elastic demand.
- If tastes are more spread out, you’ll see smaller responses.
- At the extreme in which everyone is the same, demand will be a step function, so there is some price above which no one will buy and below which everyone will buy.
- In this case, things will be very inelastic at high prices, but very elastic near the price, and then unresponsive at very low prices.
- Thinking about consumer choice in this way will be helpful for evaluating how effective sales can be.
Demand if $V \sim N(\mu, \sigma)$
Demand if $V \sim U(A, B)$

- At $B$, no one buys
- At $A$, everyone buys
Appendix Recent JMP: Piyapromdee (2018)
Setup

- Rosen-Roback: one type of worker with homogeneous tastes
  - Moretti (2011) adds idiosyncratic preferences for locations
- Piyapromdee: different worker types and taste heterogeneity
  - Education level: College vs. HS
  - Gender: F vs. M
  - Age: Young vs. Old
  - Immigrant status: Immigrant vs. Native
- Each city has 4-level nested CES function producing common traded good
Housing supply in each city

Housing “rental” rate in city c and year t:

\[ R_{ct} = i_t \times CC_{ct} \times \left[ \sum_j \gamma_h H_{jct} + \sum_j L_{jct} \right]^{\gamma_c} \]

- \( i_t \) = interest rate in \( t \)
- \( CC_{ct} \) = unobserved construction cost in \( c \) at time \( t \)
- \( H_{jct} \) = number of high education workers in subgroup \( j \), \( c \) and \( t \)
- \( L_{jct} \) = number of low education workers in subgroup \( j \)
- \( j \in \{ \text{immigrants/natives, young/old, F/M} \} \)
- \( \gamma_h = 1.68 \) is a scale factor
- \( \gamma_c \) = \( c \)-specific housing supply elasticity
Preferences across cities

Multinominal Logit Model (MNL) with utility:

\[ U_{ict} = \max_{Q,G} \lambda_z \log(Q) + (1 - \lambda_z) \log(G) + u_i(N_{ct}) + \sigma_z \epsilon_{ict} \]

\[ \text{s.t. } P_t G + R_{ct} Q = W^z_{ct} \]

- \( Q \) = amount of housing with price \( R_{ct} \)
- \( G \) = amount of numeraire good with price \( P_t \)
- \( z = z(i) \), where \( z \) is immig/natives \times young/old \times F/M \times edu level
- \( W^z_{ct} \) = wage earned by a person in group \( z \)
- \( \lambda_z \) = housing share parameter
- \( \epsilon_{ict} \sim \text{EV-I error with scale } \sigma_z \)
- \( u_i(N_{ct}) \) = person-specific utility assigned to “network characteristics” \( N_{ct} \), valued differently by each \( i \)
Utility maximization problem

Doing the maximization, we get

\[ U_{ict} = w^z_{ct} - \lambda_z r_{ct} + \beta_z X_{ict} + \sigma_z \epsilon_{ict} \]

- \( w^z_{ct} = \log(W^z_{ct}/P_t) \)
- \( r_{ct} = \log(R_{ct}/P_t) \)
- Assumes we can rewrite \( u_i(N_{ct}) = \beta_z X_{ict} \)

Indirect utility depends on log real wage \((w^z_{ct})\), and on the log of real housing prices \((r_{ct})\), but the weight on the real housing price depends on \(\lambda_z\)
Utility maximization problem

Renormalize the indirect utility by dividing by $\sigma_z$:

$$U_{ict} = \lambda_z^w (w_{ct}^z - \lambda_z r_{ct}) + \lambda_z^x X_{ict} + \epsilon_{ict}$$

$$= \Gamma_{ct} + \lambda_z^x X_{ict} + \epsilon_{ict}$$

- $\Gamma_{ct}^z$ is common in city $c$ at time $t$ for all people in $z$

Note that

- $\Gamma_{ct}^z$ captures all the endogenous variation in $w_{ct}^z$ and $r_{ct}$
- $X_{ict}$ captures person-specific network effects
  - E.g., person’s country of birth and shares of previous immigrants from the same country in $c$ and $t - 10$
Estimation of MNL model

Method: two-step “micro-BLP” approach:

1. Estimate a MNL for location choice for person $i$ including $\Gamma^z_{ct}$ dummies and person-specific components
2. Calculate determinants of $\Gamma^z_{ct}$ using $\hat{\Gamma}^z_{ct}$
Estimation

Estimating equation for $\hat{\Gamma}_{ct}^z$:

$$
\Delta \hat{\Gamma}_{ct}^z \equiv \hat{\Gamma}_{ct}^z - \hat{\Gamma}_{ct}^z - 10 = \lambda_w^z (\Delta w_{ct}^z - \lambda_z \Delta r_{ct}) + \Delta amenity_{ct}^z + \text{sampling error}
$$

- $\Delta amenity_{ct}^z = \text{change in the common amenity value of } c \text{ to people in } z$

Instrument $\Delta amenity_{ct}^z$ with “Bartik” shift-share IVs:

- Based on lagged industry shares in $c$ and national changes in employment in each industry
- Interacted with the 2 shifters of local housing elasticity
### Table 5: Parameter Estimates

#### A. Worker preferences

<table>
<thead>
<tr>
<th>Factor</th>
<th>High skill natives</th>
<th>Low skill natives</th>
<th>High skill immigrants</th>
<th>Low skill immigrants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage</td>
<td>4.028** (0.122)</td>
<td>3.725** (0.059)</td>
<td>1.228** (0.014)</td>
<td>0.726** (0.019)</td>
</tr>
<tr>
<td>Implied Rent</td>
<td>-1.208</td>
<td>-1.341</td>
<td>-0.367</td>
<td>-0.247</td>
</tr>
</tbody>
</table>

#### B. Elasticity of Substitution

<table>
<thead>
<tr>
<th>Factor</th>
<th>Estimate</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_E$: skill level</td>
<td>2.576** (0.577)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_G$: gender</td>
<td>1.924** (0.591)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_A$: age</td>
<td>8.315** (2.701)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{M-H}$: high-skill nativity</td>
<td>12.903** (2.480)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{M-L}$: low-skill nativity</td>
<td>19.928** (4.165)</td>
<td></td>
</tr>
</tbody>
</table>

#### C. Housing Supply Elasticities

<table>
<thead>
<tr>
<th>Factor</th>
<th>Estimate</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land regulation</td>
<td>3.368** (0.079)</td>
<td></td>
</tr>
<tr>
<td>Geo. constraints</td>
<td>1.223 (1.152)</td>
<td></td>
</tr>
<tr>
<td>Base housing supply elasticity</td>
<td>1.605** (0.575)</td>
<td></td>
</tr>
</tbody>
</table>

#### D. Predicted Inverse Housing Supply Elasticities

<table>
<thead>
<tr>
<th>Factor</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.211</td>
</tr>
<tr>
<td>SD</td>
<td>0.036</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.153</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.336</td>
</tr>
</tbody>
</table>

Standard errors in parentheses, clustered by MSA. **p<0.05, *p<0.1. Wage parameter estimates represent worker’s demand elasticity with respect to local real wage in a small city. Implied rent preferences are the housing expenditure shares multiplied by worker’s demand elasticity with respect to local real wage.
Estimates of $N_{ct}$ for natives

Table 6: Network Effects for Natives

<table>
<thead>
<tr>
<th></th>
<th>Young male high skill natives</th>
<th></th>
<th>Young female high skill natives</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Birth state</td>
<td>2.947**</td>
<td>2.864**</td>
<td>3.086**</td>
<td>3.063**</td>
</tr>
<tr>
<td></td>
<td>(5.0E-6)</td>
<td>(5.8E-6)</td>
<td>(8.4E-6)</td>
<td>(5.7E-6)</td>
</tr>
<tr>
<td>Distance (1000 miles)</td>
<td>-0.631**</td>
<td>-0.648**</td>
<td>-0.582**</td>
<td>-0.632**</td>
</tr>
<tr>
<td></td>
<td>(3.8E-6)</td>
<td>(4.2E-6)</td>
<td>(4.4E-6)</td>
<td>(4.3E-6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Old male high skill natives</th>
<th></th>
<th>Old female high skill natives</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Birth state</td>
<td>2.598**</td>
<td>2.512**</td>
<td>2.82**</td>
<td>2.437**</td>
</tr>
<tr>
<td></td>
<td>(1.1E-5)</td>
<td>(7.0E-6)</td>
<td>(7.8E-6)</td>
<td>(1.3E-5)</td>
</tr>
<tr>
<td>Distance (1000 miles)</td>
<td>-0.767**</td>
<td>-0.781**</td>
<td>-0.617**</td>
<td>-0.925**</td>
</tr>
<tr>
<td></td>
<td>(9.5E-6)</td>
<td>(6.2E-6)</td>
<td>(5.3E-6)</td>
<td>(1.3E-5)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Young male low skill natives</th>
<th></th>
<th>Young female low skill natives</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Birth state</td>
<td>3.808**</td>
<td>3.82**</td>
<td>3.92**</td>
<td>3.482**</td>
</tr>
<tr>
<td></td>
<td>(7.6E-6)</td>
<td>(9.8E-6)</td>
<td>(1.1E-5)</td>
<td>(6.5E-6)</td>
</tr>
<tr>
<td>Distance (1000 miles)</td>
<td>-0.556**</td>
<td>-0.524**</td>
<td>-0.506**</td>
<td>-0.771**</td>
</tr>
<tr>
<td></td>
<td>(7.0E-6)</td>
<td>(6.5E-6)</td>
<td>(7.0E-6)</td>
<td>(8.8E-6)</td>
</tr>
</tbody>
</table>
Estimates of $N_{ct}$ for immigrants

<table>
<thead>
<tr>
<th></th>
<th>Young male high skill immigrants</th>
<th>Young female high skill immigrants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of previous immigrants from same country (in million)</td>
<td>2.273**</td>
<td>1.402**</td>
</tr>
<tr>
<td></td>
<td>(1.0E-4)</td>
<td>(4.8E-5)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Old male high skill immigrants</th>
<th>Old female high skill immigrants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of previous immigrants from same country (in million)</td>
<td>2.485**</td>
<td>1.683**</td>
</tr>
<tr>
<td></td>
<td>(2.6E-4)</td>
<td>(1.0E-4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Young male low skill immigrants</th>
<th>Young female low skill immigrants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of previous immigrants from same country (in million)</td>
<td>2.445**</td>
<td>1.601**</td>
</tr>
<tr>
<td></td>
<td>(2.9E-5)</td>
<td>(1.4E-5)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Old male low skill immigrants</th>
<th>Old female low skill immigrants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of previous immigrants from same country (in million)</td>
<td>3.061**</td>
<td>1.809**</td>
</tr>
<tr>
<td></td>
<td>(9.0E-5)</td>
<td>(2.6E-5)</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. **p<0.05, *p<0.1.